STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2008–09 & thereafter)

SUBJECT CODE: MT/MC/VA32

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS
TIME : 2½ HOURS **MAX. MARKS : 100**

SECTION – A **(10 X 2 = 20)**
ANSWER ANY TEN QUESTIONS

1. If $\vec{A} = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$, find $\frac{d}{dt}\left(\vec{A}.\vec{A}\right)$.
2. Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin t, y = 2\cos 3t, z = 8t$ at any time $t > 0$.
3. Define divergence and curl of a vector point function.
4. Prove that Gradient of a constant is zero.
5. If $\phi(x, y, z) = 3x^2y - y^3z^2$,find $\nabla\phi$ at the point (1,-2,-1)
6. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is solenoidal.
7. If $\vec{f} = (t - t^2)\hat{\imath} + 2t^3\hat{\jmath} - 3\hat{k}$, find $\int_1^2 \vec{f}\,(t)dt$.
8. Show that $\vec{F} = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ is a conservative field.
9. Define surface integral of a vector point function.
10. State Gauss divergence theorem.
11. Show that $\iint_S curl\vec{F} \cdot \hat{n}dS = 0$ where S is any closed surface.
12. Show that $\int_C \vec{r}\,.\overrightarrow{dr} = 0$, where C is a closed curve.

SECTION – B **(4X20=80)**
ANSWER ANY FOUR QUESTIONS

13. a) Find an equation for the tangent plane to the surface $z = x^2 + y^2$ at the point (1,-1,2) (10)
    b) Find a unit tangent vector to any point on the curve $x = a\cos\omega t, y = a\sin\omega t, z = bt$ where a, b, ω are constants. (6)
    c) If $\vec{A}$ and $\vec{B}$ are differential functions of a scalar u, prove that
    $$\frac{d}{du}(\vec{A}\times\vec{B}) = \vec{A}\times\frac{d\vec{B}}{du} + \frac{d\vec{A}}{du}\times\vec{B}$$ (4)

14. a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction $2\vec{i} - \vec{j} - 2\vec{k}$. In which direction the directional derivative will be maximum and what is its magnitude . Also find a unit normal to the surface $x^2yz + 4xz^2 = 6$ at the point (1,-2,-1). (12)

b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 0$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2) (8)

15. a) Prove: $\nabla \times (\nabla \times \vec{A}) = -\nabla^2 \vec{A} + \nabla(\nabla \cdot \vec{A})$ (10)

b) Prove that $\nabla^2 (\frac{1}{r}) = 0$ (10)

16. a) If $\vec{F} = (2x^2 + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$, evaluate $\int_C \vec{F} \cdot dr$ around the triangle ABC whose vertices are A(0,0);B(2,0);C(2,1). (10)

b) Evaluate $\int_V (2x + y)dV$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the plane $x=0$, $y=0$, $y=2$ and $z=0$. (10)

17. Verify Stoke's theorem for a vector field defined by $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ in the rectangular region in the XOY plane bounded by the lines $x=0$, $x=a$, $y=0$ and $y=b$. (20)

18. Verify Gauss divergence theorem for the function $\vec{F} = y\hat{\imath} + x\hat{\jmath} + z^2\hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z=0$ and $z=2$. (20)