STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008–09 & thereafter)

SUBJECT CODE : MT/MC/RD54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : RANDOM VARIABLES AND THEORETICAL DISTRIBUTION
TIME : 3 HOURS MAX. MARKS : 100

ANSWER ANY SIX QUESTIONS: (EACH QUESTION CARRIES EQUAL MARKS)

1. a) State addition theorem of probability.
   b) A problem in statistics is given to three students A, B and C where chances of solving are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them try independently.
   c) State Tchebychev's inequality. Two unbiased dice are thrown. If X is the sum of the numbers showing up, then using Tchebychev's inequality prove that,
   $P\{|X-7| \geq 3\} \leq \frac{35}{54}$ (2+5+10)

2. a) State product theorem of Probability.
   b) State and prove Baye's theorem.
   c) The contents of three urns U_1, U_2, U_3 are as follows:
      1 white, 2 red, 3 black balls
      2 white, 3 red, 1 black balls
      3 white, 1 red, 2 black balls
   An urn is chosen at random and 2 balls are drawn at random. The two balls are one red and one white. What is the probability that they come from the second urn U_2 ? (2+7+8)

3. a) Define a random variable.
   b) A random variable X has the following distribution.

X	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

   (i) Determine the value of a.
   (ii) Find $P(X<3)$, $P(x \geq 3)$, $P(0<x<5)$.
   (iii) What is the smallest value of x for which $(X \leq x) > 0.8$?
   (iv) Find out the distribution function of X.

   c) Define a continuous random variable. State any two properties of probability distribution function of a random variable. For the following density function $f(x) = x^2(1-x), 0<x<1$ find the constant '*c*' and mean. (2+7+8)

4. a) Define independent random variables.

   b) Let the joint p.d.f of X_1 and X_2 be $f(x_1, x_2) = \begin{cases} c(x_1\, x_2 + e^{x_1}); 0 < (x_1, x_2) < 1 \\ 0 \quad ; \text{elsewhere} \end{cases}$

      (i) Determine c.

      (ii) Examine whether X_1& X_2 are stochastically independent.

   c) Given the following bivariate probability distribution obtain

      (i) Marginal distributions of X and Y.

      (ii) The conditional distribution of X given $Y = 2$.

Y \ X	-1	0	1
0	$^1/_{15}$	$^2/_{15}$	$^1/_{15}$
1	$^3/_{15}$	$^2/_{15}$	$^1/_{15}$
2	$^2/_{15}$	$^1/_{15}$	$^2/_{15}$

(2+7+8)

5. a) Define expectation of a random variable.

   b) State the multiplication theorem of expectation.

   c) Two random variables X and Y have the following probability density function.

   $f(x, y) = \begin{cases} 2 - x - y; \;\; 0 \le x \le 1, 0 \le y \le 1 \\ 0 \quad ; \text{ elsewhere} \end{cases}$

   Find (i) Marginal Probability density functions of X & Y.

   (ii) Var(X) and Var(Y).

   (iii) Covariance between X and Y. (3+3+11)

6. a) Define moment generating function of a random variable. Let the random variable X assume value $'r'$ with the probability law:

   $P(X = r) = q^{r-1}p; \;\; r = 1,2,,3, \ldots$

   Find the m.g.f. of X and hence its mean and variance.

   b) A fair coin is lossed four times. Let X denote the number of heads occuring and let Y denote the longest string of heads occuring.

      (i) Determine the joint distribution of X and Y.

      (ii) Find Cov(X, Y). (2+8+7)

7. a) State the additive property of Binomial distribution.

   b) Derive the moment generating function of the Poisson distribution. Find its mean, variance, β_1 & β_2. (2+15)

8. a) Derive the recurrence relation for the moments of a Binomial distribution.

   b) State any five important characteristics of the normal distribution.

   c) Obtain the median of the normal distribution. (5+5+7)

⅄⅄⅄⅄⅄⅄⅄