**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2008 – 09 & thereafter)**

**SUBJECT CODE : MT/MC/RA54**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2011**
**BRANCH I - MATHEMATICS**
**FIFTH SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : REAL ANALYSIS**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**ANSWER ANY SIX QUESTIONS**

1. a) *f(x)* and *g(x)* are real valued function if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then prove that $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$.
   b) P.T. $\lim_{x\to\infty}\left(\frac{1}{x^2}\right) = 0$. (10+7)

2. a) Let $f$ be a non decreasing function on the bounded open interval $(a, b)$. If $f$ is bounded above on $(a, b)$ then $\lim_{x\to b-} f(x)$ exist. Also if $f$ is bounded below on $(a, b)$ then $\lim_{x\to a+} f(x)$ exist.
   b) If $f$ and $g$ are real valued function, if $f$ is continuous at $a$, $g$ is continuous of $f(a)$ then $g \circ f$ is continuous at $a$. (10+7)

3. a) Define convergent sequence is metric space. In a metric space $(S, d)$ assume $x_n \to p$ and let $T = \{x_1, x_2, \dots\}$ be the range of $\{x_n\}$ then
   (i) $(T)$ is bounded (ii) $p$ is an adherent point of $T$.
   b) Define Cauchy sequence. Show that in Euclidean space $R^k$ every cauchy sequence is convergent. (9+8)

4. a) In any metric space $(S, d)$ every compact subset $T$ is complete.
   b) Let $f: S \to R^k$ be a function from a metric space $S$ to Euclidean space $R^k$. If $f$ is continuous on a compact subset $X$ of $S$, then $f$ is bounded in $X$. (10+7)

5. a) State and prove Bolzano's theorem.
   b) State and prove Fixed point theorem. (7+10)

6. a) Every open connected set in $R^n$ is arcwise connected.
   b) Let $\mathcal{F}$ be a collection of connected subjects of a metric space $S$ such that $T = \bigcap_{A\in\mathcal{F}}$ is not empty. Then $\bigcup_{A\in\mathcal{F}} A$ is connected. (9+8)

7. a) Let $f$ be a bounded function in the closed bounded interval $[a, b]$. Then $f \in \mathbb{R}[a, b]$ iff, for each $\varepsilon > 0$ there exists a subdivision $\sigma$, of $[a, b]$ such that $U[f, \sigma] < L[f, \sigma] + \varepsilon$
   b) State and prove chain rule in derivatives. (7+10)

8. a) State and prove the Law of mean.
   b) State and prove first fundamental theorem of Calculus. (7+10)