STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09 & thereafter)

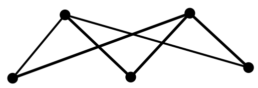
SUBJECT CODE : MT/MC/GT33

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
THIRD SEMESTER

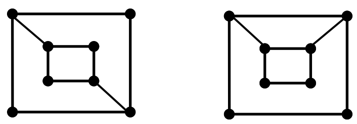
COURSE : MAJOR –CORE
PAPER : INTRODUCTION TO GRAPH THEORY
TIME : 2½ HOURS **MAX. MARKS : 100**

SECTION – A **(10 X 2 = 20)**
ANSWER ALL QUESTIONS

1. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$ for a graph $G(p, q)$.
2. Define block of a graph with an example.
3. Define adjacency matrix of a graph.
4. Prove that connectedness of points is an equivalence relation on the set of points of G.
5. Define centre of a graph G.
6. State four colour theorem.
7. Define Hamiltonian graph.
8. Is the following graph planar? If so draw it without any crossings.



9. Are the following two graphs isomorphic? Justify.



10. State True or False. a. Petersen graph is Eulerian.
b. In a tree, every vertex is a cut-vertex.

SECTION – B
ANSWER ANY FIVE OF THE FOLLOWING **(5x8=40)**

11. Draw the graph corresponding to the following incidence matrix.

$$\begin{vmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{vmatrix}$$

12. Prove that a graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line of G joining a vertex of V_1 to a vertex of V_2.
13. Show that if every block of a connected graph G is Eulerian, then G is Eulerian.
14. Prove that every tree had a centre consisting of either one point or two adjacent points.

15. State and prove Euler's theorem.
16. Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.
17. Prove that a graph G with atleast two vertices is bipartite iff all its cycles are of even length.
18. Prove that a closed walk of odd length contains a cycle.

SECTION – C

ANSWER ANY TWO OF THE FOLLOWING (2X20=40)

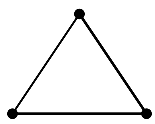
19. Prove that the following statement are equivalent for a graph $G(p, q)$.
    (i) G is a tree.
    (ii) Every two vertices of G are joined by a unique path.
    (iii) G is connected and $p = q + 1$.
    (iv) G is acyclic and $p = q + 1$.

20. a) State and prove the five colour theorem.
    b) Define the chromatic number and chromatic index of a graph and find the same for the following graphs: (i) Petersen graph (ii) $K_{m,n}$ (iii) C_{2n+1} (10+10)

21. a) Define the union and composition $\{G_1[G_2]$ & $G_2[G_1]\}$ of two graphs G_1 and G_2 and find the same for the following graphs:

G_1:  G_2: 

    b) Define product of two graphs G_1 and G_2 and find the same for the following graphs:

G_1:  G_2: 

    c) Define SUM of two graphs G_1 and G_2 and find the same if $G_1 = K_1$ and $G_2 = C_n$

(10+5+5)