STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008–09 & thereafter)

SUBJECT CODE : MT/MC/AS54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRAIC STRUCTURES
TIME : 3 HOURS **MAX. MARKS : 100**

ANSWER ANY SIX QUESTIONS

1. a. State and prove Lagrange's Theorem.

   b. If G is a finite group and $a \in G$, then prove that $O(a)$ divides $O(G)$.

   c. Prove that intersection of two normal subgroups of a group is normal.

(8+4+5)

2. a. Prove that the subgroup N of G is normal if and only if every left coset of N in G is a right coset of N in G.

   b. Prove that HK is a subgroup of G if and if only if $HK=KH$.

(8+9)

3. a. State and prove fundamental theorem on homomorphism of groups.

   b. Define normalizer $N(a)$ of an element a is a group G. Prove that $N(a)$ is a subgroup of G.

   c. Prove that the centre $Z(G)$ of a group G is normal.

(8+4+5)

4. a. If G is a group, then prove that $A(G)$,the set of all automorphisms of G is a group.

   b. State and prove Cayley's theorem.

(8+9)

5. a. Find inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 1 & 6 & 2 & 5 \end{pmatrix}$

   b. Prove that intersection of two ideals of a ring is again an ideal.

   c. Prove that any field is an Integral domain.

(5+6+6)

6. a. Prove that a finite integral domain is a field.

   b. If R is a commutative ring and $a \in R$. Then prove that $aR = \{ar/r \in R\}$ is a two sided ideal of R.

   c. If U is an ideal and $1 \in U$, then prove that $U=R$.

(6+6+5)

7. a. If ϕ is a ring homomorphism of R_1 into R_2, define kernel of ϕ. Prove that ker ϕ is an ideal of R_1.

   b. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is maximal ideal if and only if $\frac{R}{M}$ is a field.

(7+10)

8. a. Prove that every integral domain can be imbedded in a field.

   b. State and prove Division Algorithm for polynomials.

(7+10)