SUBJECT CODE : 11MT/MC/DC14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE   : MAJOR – CORE
PAPER   : DIFFERENTIAL CALCULUS
TIME   : 3 HOURS   9 MAX. MARKS : 100

SECTION – A   (10X2=20)
ANSWER ALL THE QUESTIONS

1. Find the nth derivative of $\dfrac{1}{(2x+3)^2}$.

2. If $y = ae^{mx} + be^{-mx}$, show that $y_2 = m^2 y$.

3. If $u = x^3 + y^3 + z^3 + 3xyz$, prove tha $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3u$.

4. Find $\dfrac{du}{dx}$ when $u = x^2 + y^2$, where $y = \dfrac{1-x}{x}$.

5. Find the coordinates of the centre of curvature of the curve $xy = 2$ at the point $(2,1)$.

6. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1,1)$?

7. Prove that $f(x) = x^3 - 3x^2 + 5x + 11$ is neither maximum nor minimum for any value of x.

8. Divide a given number into 2 parts such that the product of one part with the cube of the other is a maximum.

9. When is a curve said to be symmetrical

      (i)   about the x-axis   (ii)   about the y-axis

10. When is a *cusp* said to be of the first or second kind?

SECTION – B   (5X8=40)
ANSWER ANY FIVE QUESTIONS

11. Find the nth derivative of $y = \dfrac{1}{1-5x+6x^2}$.

12. If $u = \log\dfrac{x^4+y^4}{x-y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$.

13. If $u = sin^{-1}(x - y)$, $x = 3t$, $y = 4t^3$ show that $\dfrac{du}{dt} = \dfrac{3}{\sqrt{1-t^2}}$.

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14. Find the radius of curvature of the curve $xy = c^2$ at (x, y).

15. Find the coordinates of centre of curvature for any point (x, y) on the parabola $y^2 = 4ax$.

16. Determine the maximum and minimum value of the following function
$x^5 - 5x^4 + 5x^3 - 1$.

17. Trace the curve $r = a + b \cos \theta$.

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SECTION – C   **(2X20=40)**
ANSWER ANY TWO QUESTIONS

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18. a) If $y = a \cos(\log x) + b \sin (\log x)$, show that
$$x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0.$$
   b) I $u = \log \dfrac{x^2 + y^2}{xy}$, prove that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$.

19. a) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(a/4, \ a/4\right)$.

   b) Discuss the maximum and minimum of the function $x^3 y^2 (6 - x - y)$.

20. a) Trace the curve Cissoids of Diocles.

   b) Trace the curve $r^2 = a^2 \cos 2\theta$. What is this curve known as ?

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(10+10)

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