STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2011–12)

SUBJECT CODE : 11MT/MC/AT14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRA AND TRIGONOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A (10X2=20)

ANSWER ALL THE QUESTIONS

1. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 3 & 4 & 3 \end{bmatrix}$.
2. State the condition for consistency of a system of linear equations.
3. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.
4. Find the equation with rational coefficients, given one root is $1 + i\sqrt{3}$.
5. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, find the value of $\sum \alpha^2\beta$.
6. Expand $\sin 5\theta$.
7. Prove that $\sinh 3x = 3 \sinh x + 4sinh^3x$.
8. Express $tanh^{-1}x$ interms of logarithmic function.
9. Separate into real and imaginary parts $\cos(x - iy)$.
10. Find $Log_e(\sqrt{3} + i)$.

SECTION – B (5X8=40)

ANSWER ANY FIVE QUESTIONS

11. For what values of k the system of equations $kx + 2y - 2z = 1$, $4x + 2ky - z = 2$, $6x + 6y + kz = 3$ has (i) no solution (ii) a unique solution (iii) infinite number of solutions.

12. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

13. Find the equation whose roots are the squares of the roots of
$$x^4 + x^3 + 2x^2 + x + 1 = 0.$$

14. Prove that $\frac{\sin 7\theta}{\sin \theta} = 7 - 56sin^2\theta + 112sin^4\theta - 64sin^6\theta.$

15. Expand $sin^4\theta\, cos^2\theta$ in a series of cosines of multiples of θ.

16. If $\sin(\theta + i\varphi) = \tan(x + iy)$, prove that $\frac{\tan\ \theta}{\tanh\ \varphi} = \frac{\sin\ 2x}{\sinh\ 2y}$.

17. (i) Separate $tan^{-1}(x + iy)$ into real and imaginary parts.

(ii) If $\cosh u = \sec\theta$, prove that $u = log\left[tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. a) Determine the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using its characteristic equations.

b) Diagonalise the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$. (7+13)

19. a) If α, β, γ are the roots of $x^3 + 2x^2 + 3x + 3 = 0$, prove that $\sum \frac{\alpha^2}{(\alpha+1)^2} = 13$.

b) Determine completely the nature of the roots of the equation

$x^5 - 6x^2 - 4x + 5 = 0.$

c) Solve $x^4 + 2x^2 - 16x + 77 = 0$, if one root is $-2 + \sqrt{-7}$.

(8+8+4)

20. a) Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{\sin\ x + \cos\ 2x}{cos^2 x}$.

b) Prove that one value of $\alpha + i\beta = b^{x+iy}$ is $\frac{tan^{-1}\left(\beta/\alpha\right)}{log_e(\alpha^2+\beta^2)}$.

c) If $\cos\alpha \cosh\beta = cos\varphi$, $\sin\alpha\sinh\beta = sin\varphi$ prove that

$\sin\varphi = \pm sin^2\alpha = \pm sinh^2\beta$ (6+7+7)

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