STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011-12)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2011
BRANCH IV - CHEMISTRY
FIRST SEMESTER

COURSE   : ALLIED – CORE
PAPER   : MATHEMATICS FOR CHEMISTRY – I
TIME   : 3 HOURS   MAX. MARKS : 100

SECTION – A   **(10 X 2 = 20)**
ANSWER ALL THE QUESTIONS

1. Define characteristics roots of a matrix.

2. Show that the two matrices A and $P^{-1}AP$ have the same characteristic roots.

3. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rs + s = 0$ then find S_1, S_2, S_3, S_4.

4. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that $1 + \sqrt{-1}$ is a root of it.

5. Prove that $cosh^2 x + sinh^2 x = \cosh 2x$.

6. Find $\dfrac{dy}{dx}$ when x and y are connected by the relation $x^2 + y^2 = a^2$.

7. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x+y+z}$.

8. If $u = \dfrac{xy}{x+y}$ show that $x\dfrac{\partial u}{\partial x} + +y\dfrac{\partial u}{\partial y} += u$.

9. Evaluate $\int sin^2 3x \ dx$.

10. Prove that $\int_0^{\frac{\pi}{2}} sin^n x \ dx = \int_0^{\frac{\pi}{2}} cos^n x \ dx$.

SECTION – B   **(5 X 8 = 40)**
ANSWER ANY FIVE QUESTIONS

11. Find all the characteristic roots of the matrix
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}.$$

12. Find the characteristics equation of $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ and show that the matrix A satisfies the equation.

13. Solve the equation $x^3 - 19x^2 + 114x - 216 = 0$ given that the roots are in G.P.

14. If $x = \sin\theta$, $y = \cos p\theta$, prove that $(1 - x^2)y_2 - xy_1 + p^2y = 0$.

15. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x}}}\,...$ to infinity find $\dfrac{dy}{dx}$.

16. If $z = f(x, y)$ and $x = a\cos\theta$, $y = s\sin\theta$, prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

17. Find $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$.

SECTION – C   (2 X 20 = 40)
ANSWER ANY TWO QUESTIONS

18. Diagonalise the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

19. a) If α, β, γ are the roots of the equation $x^3 + px^2 + s = 0$, find the value of $\alpha^3 + \beta^3 + \gamma^3$.
   b) Solve the equation $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

20. (i) Find the nth differential coefficient of $\cos x, \cos 2x, \cos 3x$.

   (ii) If $u = (y - z)(z - x)(x - y)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

   (iii) Evaluate $\displaystyle\int \frac{dx}{1-6x-9x^2}$.

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