CHAPTER I

INTRODUCTION

1.1 Two-phase Flows and Their Importance

(Fluid flows with particulate suspensions, when the suspended matter may consist of solid particles, liquid droplets, gas bubbles or combinations of these, are commonly termed dusty gas flows or dusty fluid flows. They are also referred to as two-phase flows, since they involve a composite of two phases or two materials with different distinguishable properties - one phase being the fluid medium which is a continuous phase and the other phase being the particulate suspensions which are scattered throughout the fluid medium and hence known as the dispersive phase or discrete phase or simply particulate phase. The fluid medium can be either gas or liquid.) When the fluid medium is a gas, the particulate phase may consist of solid particles or liquid droplets or both. When the fluid medium is a liquid, the particulate phase may consist of solid particles, gas bubbles or liquid droplets which are immiscible to the fluid phase. In general, a multi-phase flow consists of a fluid phase or fluid medium and a particulate phase of any number of chemical components.

The flows of fluids with suspended material particles abound in nature, classical examples being pollution of air and contamination of water. The earth's atmosphere is a predominantly gaseous envelope of air surrounding the earth and it contains solid particles and liquid droplets. Besides it is also being constantly polluted by a number of dust particles like carbon suit, sulphur and many other toxic elements which arise as inevitable consequences and natural by-products of rapid industrialization. The natural waters around us are contaminated by sewage solids which are being dumped into seas, rivers and harbours. Industrial activity, especially pulp and paper production, food processing and chemical manufacturing, generates a wide variety of waste products that are being discharged into flowing waters. Thus new types of wastes continue to appear as new technology develops and these find entry into the earth's atmosphere or into the natural waters of seas, rivers and lakes. In order to minimise and control these grave problems of pollution of air and water a knowledge of the behaviour of two-phase flows or multi-phase flows is necessary.

Problems of two-phase flows arise in many engineering and industrial applications. In heat transfer technology, fluids embedded with dust particles are used in gas-cooling chambers to enhance heat transfer processes, as it is well known that solid particles are better conductors of heat than liquids. Dusty gas flows assume importance in such engineering problems as fluidization (flows through packed beds), sedimentation, powder technology, flows in rocket tubes where small carbon or metallic fuel particles are present, aerosol filtration, gas purification, motion of suspension and slurries, and in the process by which raindrops are formed by the coalescence of small droplets which might be considered as solid particles for the purpose of examining their movement prior to coalescence. Similar situations can also be noticed in the flows of fluids containing dissolved micro-molecules, fiber suspensions, latex particles in emulsion paints, reinforcing particles in polymer melts and rock crystals in molten lava. A knowledge of two-phase flows is of vital importance in petroleum industry and in the purification of crude oils. Further, problems concerned with atmospheric fall out, batch settling, rain erosion of guided missiles and aircraft icing are some of the areas where the dynamics of dusty gases play a prominent role. Other important applications involving dust particles in boundary layers include soil salvation by natural winds, lunar surface erosion by the exhaust of a landing vehicle and dust entrainment in a cloud formed during a nuclear explosion. More recently, the revolutionary growth in the field of propulsion and combustion had its impact and stimulated new interest in the gas-particle flow phenomena.

A knowledge of dusty fluid flows is useful to some extent in understanding the rheology of blood flows through capillaries, where red blood cells can be regarded as rigid particles embedded in the plasma which is a Newtonian fluid. Thus by treating blood as a dusty fluid, it is possible, for example, to determine the effect of red corpuscles on the velocity distribution of the plasma and to assess the loss of pressure head in the capillaries due to the presence of red cells. These informations would help in the diagnosis of diseases connected with the circulatory system and in the design of such medical apparatuses as blood pumps and oxygenators. Another biological situation where the study of two-phase flows assumes importance is the phenomena of particle deposition in the respiratory tract.

Besides all these examples drawn from nature, science, technology and biology, recent space-craft observations have confirmed that dust particles play a prominent role in the dynamics of the Martian atmosphere. This planet is periodically swept by huge dust storms. Besides, wind-driven particles have played an important role in the shaping of the Martian surface (Pollock (1975)). Space research has also shown that Venus is covered by a dense layer of clouds consisting of suspended particles of different concentrations and sizes (Schubert and Covery (1981)). Further, scientists are always concerned about the meteoric dust hazards to space stations. Thus with the advancement of space technology, the dynamics of dusty gases has found applications in such extra-terrestrial fields as the study of other planets and also in the cosmological theories of evolution of stars and planets from dust-laden gases.

1.2 Survey of Literature

The study of the flows of fluids with suspended solid particles appears to have been initiated by Stokes (1851) who examined the resistance of a single solid body (spherical pendulam bob) moving relative to a fluid (air) in which viscosity was taken into account. The relationship he discovered is the familiar Stokes' drag law and it has been found to apply to flow situations in which a number of solid particles are present, provided the particles are far enough apart on the average that their motion is not affected by the mutual interaction of individual particles.

Later Einstein (Landau and Lifshitz (1959)) investigated the resistance to shear of a suspension of small spherical particles immersed in a continuous fluid and showed that the apparent increase in viscosity of the suspending fluid is related to the volumetric concentration of solid particles by a simple proportionality constant. The Einstein's formula for suspension viscosity has since been used to analyse the motion of suspensions in shearing fields of flow. Like Stokes' law, Einstein's formula is also applicable to the case when the suspended particles are sufficiently far apart that the motion of each one of them is not affected by the motion of others. Einstein's viscosity theory for suspension of spherical particles was extended by Jeffery (1922) to particles of ellipsoidal shape. Guth and Simha (1936) further extended Jeffery's work by considering wall effects and interaction between particles on the apparent viscosity of suspensions.

Hoenig (1957), Carrier (1958), Soo (1961) and others have done a lot of pioneering work on the effect of dust particles on shock waves. While Hoenig (1957) examined the acceleration of dust particles when a shock wave of constant strength enters a dust-laden gas, Carrier (1958) considered the effect of stationary plane shock configuration in a dusty gas. Soo (1961) formulated the basic gas dynamic equations involving suspended solid particles by taking into account momentum and heat transfer between gaseous and solid phases and applied them to normal shock problems. Later Rudinger (1964) analysed some properties of shock relaxation in gas flows carrying small particles. Special mention must be made of Marble's papers (1962, 1963) on the study of shock waves and the Prandtl-Meyer expansion in dusty gases.

(The observation that adding dust to air flowing in turbulent motion through a pipe can considerably reduce the resistance coefficient, has been reported by Sproull) (1961). A similar report that the aerodynamic resistance of a dusty gas flowing through a system of pipes is less than that of a clean gas has also been made by Kazakevich and Krapivin (1958). These observations can be expressed as saying that the pressure difference required to maintain a given volume rate of flow is reduced by the addition of dust. It may be noted that the increased density of a dusty gas should, all other things being equal, require a larger pressure difference to maintain a given volume flow rate. Sproull's interpretation that there will be reduction in the viscosity of a dusty gas as compared with a clean gas contradicts the Einstein's formula for the viscosity of a suspension, according to which the viscosity of a dusty gas should be increased by a factor proportional to the concentration by volume of the dust particles. Saffman (1962) gave a more plausible explanation by stating that dust damps the turbulence. A dust particle in air, or any other gas, has a much larger inertia than the equivalent volume of air and will not therefore participate as readily in the turbulent fluctuations. The reduction of the intensity of the turbulence leads to a reduction of the Reynolds stresses and the force required to maintain a given flow rate is likewise reduced.

Since the problem of turbulence is related to the stability of laminar flows, Saffman (1962) studied the effect of dust particles on the stability of a laminar flow of a gas, in order to see how dust may affect the critical Reynolds number for transition from laminar to turbulent flow. He derived a modified Orr-Sommerfeld equation for this purpose and showed that the addition of fine dust destabilizes a gas flow, while the addition of coarse dust has a stabilizing effect.

To describe dusty fluid flows mathematically, Saffman (1962) made some simplifying assumptions regarding the motion of dust particles in the fluid and formulated a separated flow model in which separate equations are written for each phase. (On account of its relative simplicity, Saffman's dusty gas model has opened up a new and exciting era in the field of fluid dynamics. For the past two decades there has been a spate of research activity in the area of dusty fluid flows), a brief resume of which is given below.

Michael (1964) solved numerically the Orr-Sommerfeld equation derived by Saffman, and found that the dust particles modify the neutral stability curves. Michael (1965) also studied the Kelvin-Helmoltz instability of a plane vortex sheet in a dusty gas. Further he (Michael (1968)) considered the steady motion of a sphere in a dusty gas and found that in the inviscid model there exists a dust-free layer adjacent to the sphere, while in the viscous model the dust-free layer is preserved only when $\sigma\sqrt{R} \gg 1$, where σ denotes the Stokes number and R is the radius of the sphere. Drew (1979) discussed the stability of a Stokes layer of a particle-fluid mixture taking into consideration the buoyancy force on the particle-phase and derived a set of Orr-Sommerfeld equations with modified Reynolds number. The presence of fine dust particles is found to destabilize the Stokes layer flow. In the case of coarse dust, for low mass concentration and for sufficiently large values of the particle relaxation time, it is found that the flow is unstable.

All the works on the stability of two-phase flows mentioned above are based upon linear stability analysis in which the disturbances are considered to be very small so that the non-linear terms are neglected. However Dandapat and Gupta (1976) studied the stability of flow of a dusty gas for arbitrary non-linear disturbances using the energy method due to Serrin (1959). They derived an improved universal stability estimate and showed that the presence of dust particles destabilizes the flow.

While Michael (1968) considered the flow of a dusty gas past a fixed sphere, Sambasiva Rao (1973) analysed the steady motion of a sphere in a dusty gas, otherwise at rest and showed that when a non-singular perturbation of a potential flow is assumed, the dust particle concentration becomes logarithmically infinite to the front stagnation point of the sphere. Further, he observed that the dust particles cannot reach the sphere except at the front stagnation point, there being a dust streamline emanating from the point which delineates a thin dust-free layer adjacent to the sphere. In the case of dusty fluid flow past a fixed sphere, Michael (1968) showed that the particles do not collide with the sphere until the Stokes number σ is greater than a critical value $\sigma_{crit} = 1/12$. Treating the problem of steady motion of a circular cylinder in a dusty gas, recently Rukmangadachari (1981) observed that for a circular cylinder $\sigma_{crit} = 1/8$. Michael and Norey (1970) considered the problem of slow motion of a sphere in a two-phase medium and observed that the particles which lie on the surface cannot escape from the surface. Miller (1969) treated the problem of motion and number density of particles, initially at rest, under gravity in an incompressible, viscous fluid round a vertical, solid circular cylinder in oscillatory rotation and developed methods for finding trajectories and number densities of the particles. Nirmala and Arunachalam (1978) studied the primary flow due to the rotatory oscillations of two spheres in a dusty viscous fluid using Stokes' linearization technique and bipolar coordinates and observed that for steady rotation the torque on the sphere is not affected by the dust particles.

Michael and Miller (1966) investigated the motion of a dusty fluid occupying the semi-infinite space above a rigid plane boundary when the plate executes simple harmonic oscillations and when it starts impulsively from rest with uniform velocity. The change in phase velocity and the decay of oscillatory waves are obtained as functions of the mass concentration λ. It is shown that for large time, the shear layer thickness decreases by a factor $(1 + \lambda)^{-1/2}$. Datta and Jana (1976) extended the work of Michael and Miller to the flow of a dusty gas induced by an oscillating flat plate in a rotating frame of reference. Jana and Datta (1977) also solved the problem of flow of a dusty gas over an infinite plate executing non-torsional oscillations in a rotating frame of reference. The steady-state solution reveals the existence of multiple boundary layers, the thickness of which decrease as the mass concentration parameter increases. Further the presence of dust particles eliminates the occurrence of resonance in the fluid motion. Using regular perturbation technique Nag (1980) solved the two-dimensional flow of a dusty fluid induced by sinusoidal wavy motion of an infinite wavy wall and found that the oscillations decrease rapidly along the transverse direction.

Fluid flows between parallel plates constitute an important class of problems in fluid dynamics, since they approximate to flows that are frequently encountered in many engineering disciplines. Hence a lot of work has been done in this direction for dusty fluid flows, like those of Vimala (1972), Dube and Singh (1972), Sacheti and Bhatt (1972), Sharma (1975), Singh (1977), Mathur et al (1976), Nag et al (1979), Dube and Sharma (1975), Ramana Prasad and Pattabhiramacharyulu (1980, 1981), Mitra and Bhattacharyya (1981). Mitra (1981) considered the oscillatory flow of a dusty gas between parallel plates in a rotating frame of reference. Verma and Sarangi (1981) studied the unsteady flow of a dusty fluid between two wavy walls with roughness along their length, whose equations are taken in the form of Fourier series with the assumption that ϵ , the coefficient of roughness, is small and obtained velocity profiles to the first order of ϵ .

Realising the importance of fluid flows through circular geometries, dusty fluid flows through circular pipes under different types of pressure gradient have been analysed by many researchers like Rao (1969), Tewari and Bhattacharjee (1973), Singh and Dube (1975), Kishore and Pandey (1977) and Arunachalam et al (1976). Singh and Pathak (1977, 1977) analysed the unsteady flow of a dusty fluid through a uniform tube with sector of a circle as cross-section under the influence of (i) a constant pressure gradient and (ii) an exponential pressure gradient with respect to time, and inferred that in case (i) the velocity of dusty fluid is greater than that of dust particles, while in case (ii) the velocity of dust particles is greater than that of the fluid. The general time-dependent flow of a dusty gas in cylinders with circular and sectorial cross-sections has been discussed by Gupta and Gupta (1977), when the pressure gradients are arbitrary functions of time.

Dusty fluid flows through the annulus between two infinite coaxial cylinders has attracted the attention of many investigators. Michael and Norey (1967) studied the laminar flow of a dusty gas between two rotating cylinders, by assuming the relaxation time of dust to be small and considering the ratio of the time scales, on which the gas velocity and the mass concentration of dust change, to be both large and small. They obtained solutions under various boundary conditions. Pathak and Upadhyay (1981) observed that the flow of a dusty gas between two rotating coaxial cylinders is always stable if the Rayleigh's criterion is satisfied. Further they discussed the principle of exchange of stabilities and solved the characteristic value problem under the narrow-gap approximations. Other significant contributions are those by Girishwar Nath (1970), Devi Singh (1973), Gupta and Gupta (1975), Rukmangadachari (1978) and Mitra (1979). Gupta and Sharma (1978) analysed the unsteady flow of a dusty viscous fluid through long confocal elliptical ducts under an arbitrary time-varying axial pressure gradient, by repeated use of the finite Mathieu transform and the Laplace transform.

Quite an amount of extensive work has been done in dusty fluid flows through pipes of different geometries. In this connection mention may be made of Crooke and Walsh (1974) who studied the flow through an infinitely long pipe with rectangular, circular and arbitrary cross-sections, Janaki Raja et al (1978) who dealt with flow between parallel plates, flow through an equilateral triangular duct and flow through an elliptic duct, under a periodic pressure gradient, Arunachalam (1979) who analysed the flow through a cylinder of triangular cross-section under a sinusoidal pressure gradient and an exponential pressure gradient and Rukmangadachari (1981) who considered the flow through a cylinder of rectangular cross-section under time-dependent pressure gradient.

There are some authors who considered other types of problems. Zung (1969) investigated the flow of a fluid-particle suspension over an infinitely large disk rotating with a constant angular velocity. Sone (1972) considered the steady flow past a body fixed in a uniform flow of a dusty gas. Crooke (1976) developed uniqueness criteria for the solution of the steady state flow problem with Saffman's model when the number density of the particles is constant and found that the criteria depends on the physical parameters of the system. Surendra Prasad (1979) developed techniques for the measurement of film thickness in annular two-phase gas-liquid flows and Gupta (1979) studied the unsteady Hele-Shaw flow of a dusty viscous fluid. Palaniswamy and Purushotham (1981) showed how the stability of shear flow of stratified fluids with fine dust is dependent on the value of the local Richardson number. Rukmangadachari (1979, 1983) considered the dusty viscous flow due to torsional vibrations of a disc and also the problem of viscous drainage on a vertical flat plate in a two-phase medium. In the latter case he found that due to the presence of dust there is an increment in the inertial contribution to the film thickness, which is proportional to the mass concentration of dust.

The brief survey given above reveals that the work done in dusty gas flows on the basis of Saffman's model comprises mainly of unsteady rectilinear problems, save the studies of steady motion of a sphere by Michael (1968, 1970) and Sambasiva Rao (1973). As can be seen from the

Saffman's equations, a study of the steady dusty viscous flow problems under the usual Stokes' linearization scheme by completely neglecting the convective terms from the equations, cannot be done, since in such a case both the fluid velocity and dust particle velocity become equal and the solutions become trivial. So to tackle steady flow problems, the non-linear equations have to be linearized by perturbation techniques such as the one used by Michael (1968), or they have to be solved by using some numerical methods.

Boundary layer theory for a dusty gas past an infinite plate in the primary stages of the motion has been initiated by Chakrabarti (1971). Gupta and Pop (1975) solved the initial value problem of unsteady boundary layer flow generated in a dusty viscous liquid bounded by an infinite flat plate when both the liquid and the plate are initially in a state of rigid body rotation about an axis normal to the plate and then the plate is impulsively started with a uniform velocity in its own plane. Further the general features of the unsteady boundary layer with particular reference to the effect of rotation through inertial oscillations and the establishment of Ekman boundary layer are discussed by them. Datta and Nag (1979) studied the boundary layer flow of a dusty gas past a flat plate when a small oscillatory flow is superimposed on the free stream flow. The velocity fields for the gas and particle-phase are separated into steady and unsteady parts. The steady state part of the solution implies that the boundary layer thickness increases with increase of either the mass concentration or the size of the dust particles. Srivastava et al (1980) investigated the boundary layer of a density stratified fluid with a suspension of particles, when the motion in the fluid, occupying the semi-infinite region above a flat plate, is induced by potential flow. Datta and Mishra (1980) used Karman-Pohlhausen method to solve the two-dimensional stagnation point flow of a dusty fluid near an oscillating plate and found that the boundary layer thickness increases when the mass concentration of dust increases or when the relaxation time decreases.

Most of the literature on the subject of dusty gas flows restricts discussion to uniform dust particle distribution. Allowing the dust particle distribution function, i.e. the number density of dust particles N, to be variable, Barron (1977) considered the case where the velocity of the dust particles are everywhere parallel to that of the fluid. He showed that the only possible flows are radial flow and flow in parallel straight lines and concluded that in the case of radial flow the dust particle distribution cannot be uniform as is possible in the case of flow in parallel lines. Following Barron, Purushotham and Reddy (1978) discussed the kinetic and kinematic properties of plane viscous dusty fluid flows.

Very few authors paid attention to the discussion of the dissipation of energy or heat transfer in two-phase flows. Studying the flow induced by an oscillating flat plate in a dusty gas, Liu (1966) not only obtained the gas and particle velocity profiles and found the shear stress on the plate but also discussed the mechanical energy dissipation. Ahmadi and Shahinpur (1974) investigated the decay of the kinetic energy of a dusty gas.

Attempts were also made to study the magnetic effects on the dusty fluid flows. Baral (1968), who considered the plane parallel flow of a conducting dusty gas, appears to be the first to have made such an attempt. In formulating the problem, the fluid is assumed to be electrically conducting, while the dust particles are non-conducting. Since the solution given by Baral was incorrect, Ramana Rao and Ramamurthy (1972) gave a correct approximate solution to the problem when the magnetic lines of force are fixed relative to the fluid and subsequently, in another note Ramana Rao (1973) solved the same problem when the magnetic lines of force are fixed relative to the plate. Ramana Rao and Kama Sastry (1973) by iteration on the mass concentration of dust, solved also the problem of flow past a plate. Yang and Healy (1973) analysed the Stokes problems of a dusty, conducting fluid over an infinite plate, set into motion in its plane by impulse and by oscillation, in the presence of a transverse magnetic field. Other significant studies in the area of dusty conducting fluid flows are those by Singh (1977), Dubey (1978), Dixit (1978), Agrawal et al (1979), Gupta and Agrawal (1980), Guha (1981), Nirmala (1981), Sastry and Seetharamaswamy (1982).

Much earlier to the studies mentioned above, Nayfeh (1966) investigated the oscillating two-phase flow through a rigid circular pipe, by formulating the equations for the two-phase motion taking the volume-fraction of the dust particles into account. Nag and Jana (1981) extended the work of Nayfeh by considering an axial wave under pulsative pressure and assuming the tube to be elastic. Using Nayfeh's model, recently Datta and Nag (1981) studied the flow of a dusty fluid in a rectangular channel under an impulsive periodic pressure gradient, while Gupta (1981) analysed the flow of a dusty fluid through a channel of various cross-sections when the axial pressure gradient is an arbitrary function of time. But on the whole not much work seems to have been done in the area of dusty fluid flows taking the volume-fraction of dust particles into consideration.

While all the studies on dusty fluid flows enumerated so far are based on a linear dusty gas model, Drew (1976) developed a non-linear model with a non-linear lift force and the usual linear Stokes' drag force acting on the dust particles. He showed that the state of uniform fluidization, which is to impart the character of fluid-like motion to a bed of solid particles, is unstable for small disturbances due to the lift forces on the particle-phase becoming dominant during the initial growth of the small disturbances.

Much earlier, Murray (1965) developed a mathematical model to describe the phenomenon of fluidization on a continuum basis. He showed that the system is unstable to small internal disturbances and that surface waves can be propagated (with attenuation) in a composite fluid. These results are in agreement with experiment. Further hot beds, where strongly exothermic reactions may be taking place, centrifugal beds (beds fluidized within a rotating system), and electromagnetic beds (those in which the particulate phase is electrically conducting) are all shown to be unstable to small internal disturbances.

Drew (1976) constructed a simple continuum linear model for gas-particulate flows and showed that the vorticity of the particle motion enclosed by a particle streamline is equal to the fluid vorticity enclosed by that streamline. Further the net flux of fluid vorticity through a particle streamline is shown to be equal to zero. The pressure drop along a fluid streamline is related to the net drag force along that streamline. Employing Drew's model, Kaimal (1978, 1979) studied the peristaltic motion of a suspension of rigid particles in a tube of arbitrary wave shape and also the steady flow of a dilute suspension in a slowly varying axisymmetric tube at low Reynolds number. Gopalan (1983) presented an exact similar solution for the unsteady flow of a dilute suspension in a semi-infinite contracting or expanding circular pipe. Saroj Prabha and Jain (1983) investigated gas-particulate flow with incompressible gas phase and compressible particle phase through a tube of varying cross-section and developed series solution for small Reynolds number. It is found that the particulate density increases in the radial direction and becomes large at the tube wall.

Very recently Elghobashi and Abou-Arab (1983) developed a two-equation turbulence model for predicting two-phase flows. The two equations describe the conservation of turbulence, kinetic energy and dissipation rate of that energy for the carrier fluid in a two-phase flow. This new model estimates the need to simulate in an ad hoc manner the effects of the dispersed phase on turbulence structure. Preliminary testing has indicated that this model is successful in predicting the main features of a round gaseous jet laden with uniform-size solid particles.

1.3 Models for Dusty Gases

(Several mathematical models for dusty gases have been proposed and used in the literature following different approaches. These models differ widely in various details. Most of the dusty gas models are linear models in that the force of mechanical interaction between the dispersive phase and the continuous phase is assumed to be proportional to their relative velocity.)

Murray (1954) derived the basic equations for two-phase flow with finite volume-fraction of solid particles using the continuum theory. Marble (1963) made an attempt to improve the model of Murray by applying the modern techniques of fluid dynamics. He introduced the temperature and diameter of solid particles in the distribution function and framed the fundamental equations. Marble (1964) also developed a dusty gas model for particle collision process in one-dimensional flow of a gas, containing solid particles of two different sizes, in which the effect of particle collisions is accounted for as well as the interaction between the particles and the gas. Soo (1967) formulated a model with solid particles of each size as a species in the mixture. Using simple kinetic theory Pai (1973) derived the fundamental equations from Boltzmann's equations for the mixture of a gas and a pseudo-fluid of small spherical solid particles. Drew (1976) developed a non-linear model in which the mechanical interaction between the two phases is a non-linear lift force along with the usual linear Stokes' drag force. The lift force is proportional to the velocity of the particle relative to the fluid and to one-half power of the shear rate which is a generalization of a result due to Saffman (1965) on the lift force experienced by a small sphere in an unbounded shear flow of a highly viscous fluid. For this purpose he employed the general constitutive principles of equipresence, invariance of coordinate transformations, objectivity, etc. as given by Truesdell and Noll (1965) and the constitutive principles enunciated by Drew and Segel (1971) applicable to two-phase continuum. Peddieson (1976) employed a non-linear model based on ad-hoc assumptions to study the unsteady parallel flows of particulate suspensions under two-phase and multi-phase systems.

(The work presented in this thesis is based on three linear models for two-phase flows and ~~a detailed~~ presentation of ~~these models is given below~~.

(1.3.1 Saffman's Model

The simplest and perhaps the most commonly used linear model for dusty gases is the one formulated by Saffman (1962). In constructing the model Saffman (1962) supposed the dust particles to be uniform in size and shape. He further assumed that the bulk concentration (i.e. concentration by volume) of the dust, say ϕ, is very small and can be neglected, while the mass concentration of dust, say λ, is a significant fraction of unity. By Stokes' drag law the net effect of the dust on the gas is equivalent to an extra frictional force $KN(\bar{v} - \bar{u})$ per unit volume, where $\bar{u}(\bar{x}, t)$, $\bar{v}(\bar{x}, t)$ are the velocities of the fluid and dust respectively, $N = N(\bar{x}, t)$ is the number density of the dust particles and K is the Stokes' drag coefficient ($K = 6\pi\mu a$ for spherical particles of radius a, μ being the viscosity of the fluid).) Assuming that the Reynolds number of the relative motion of the fluid and dust is small compared with unity so that the force between the fluid and dust is proportional to their relative velocity and that the fluid is incompressible, the equations of conservation of momentum and of mass for the fluid phase are

$$\rho \left[\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla) \bar{u} \right] = - \nabla p + \mu \nabla^2 \bar{u} + KN(\bar{v} - \bar{u}) \quad (1.1)$$

$$\nabla \cdot \bar{u} = 0 \quad (1.2)$$

where p is the pressure and ρ is the density of the fluid.

To formulate the equations of motion for the particulate phase Saffman made some further simplifying assumptions regarding the motion of dust particles in the fluid. The minimum size of the solid particles is assumed to be large enough to contain millions of molecules so that there is no individual molecular motion within a dust particle. In the absence of such molecular motions the discrete phase does not contribute any pressure gradient forces in the field equations. Besides, the number density of the discrete phase is assumed to be sufficiently small as to neglect the effects of the Brownian motion. The force exerted on the dust by the fluid is equal and opposite to that exerted on the fluid by the dust. Further, the dust particles, due to its inertia, do not necessarily follow the streamlines of the fluid. Hence the material rates of change bear the distinction that

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i} \qquad (1.3)$$

for the fluid, while

$$\frac{D}{Dt_p} = \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} \qquad (1.4)$$

for the dust particles, where t is the time variable and x_i is the space coordinate.

Hence the equations of conservation of momentum and of mass for the dust phase are

$$mN \left[\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\, \bar{v} \right] = mN\bar{g} + KN(\bar{u} - \bar{v}) \qquad (1.5)$$

$$\frac{\partial}{\partial t}(mN) + \nabla \cdot (mN\bar{v}) = 0 \qquad (1.6)$$

where m is the mass of a dust particle and $\bar{g}$ is the acceleration due to gravity, the force of buoyancy (weight of liquid displaced by dust particles per unit volume, viz. $-\phi \rho \bar{g}$) being neglected since ϕ is small.

Further the weight of a dust particle acting vertically down is assumed to be much smaller than the drag force and hence neglected. This amounts to assuming that the suspended dust particles are not heavy enough to settle under the influence of gravity and hence the sedimentation effects are ignored. Thus equation (1.5) reduces to

$$\frac{D\vec{v}}{Dt_p} = -\frac{K}{m}(\bar{v} - \bar{u}) \tag{1.7}$$

If the number density N of dust particles is assumed to be a constant throughout the motion, equation (1.6) reduces to

$$\nabla \cdot \bar{v} = 0 \tag{1.8}$$

Because of the inertia of solid particles, a gas-solid suspension demonstrates an interesting nature of relaxation. This is a familiar concept in the phenomenon of radioactivity which is the spontaneous decomposition or disintegration of a nucleus into lighter fragments. Experimental evidence has shown that the radioactive decay follows the exponential law:

$$N(t) = N_o\, e^{-\gamma t} \tag{1.9}$$

or equivalently the differential equation

$$\frac{dN}{dt} = - \gamma N \tag{1.10}$$

where N is the number of undecayed atoms still present at time t and N_0 is the initial number at time $t = 0$, γ being a constant known as the disintegration constant. The relaxation time τ is defined as the time taken by $N(t)$ to attain $1/e$ of its value at time t and it is found to be $1/\gamma$. The equation of motion for dust viz. (1.7) resembles equation (1.10) which characterizes radioactive decay. Hence the relaxation time τ in this case is the time taken by the velocity of dust relative to the fluid to become $1/e$ of its value at time t and $\tau = m/K$.

Thus the effect of dust is characterized by the two parameters:

i) the mass concentration $\lambda = mN/\rho$ which is the ratio of the density of the dispersive phase to that of the fluid phase and is a dimensionless quantity and

ii) the relaxation time $\tau = m/K$ which is a measure of time taken by the dust particles to adjust to changes in the gas velocity and has the dimensions of time.

The former parameter describes how much dust is present and the latter parameter is determined by the size of the individual particles.

Equations (1.1) and (1.7) can be rewritten in terms of dust parameters as

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla)\,\bar{u} = -\frac{1}{\rho}\,\nabla p + \nu \nabla^2 \bar{u} + \frac{\lambda}{\tau}(\bar{v} - \bar{u}) \qquad (1.11)$$

$$\tau \left[\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\,\bar{v} \right] = \bar{u} - \bar{v} \qquad (1.12)$$

where $\nu = \mu / \rho$ is the kinematic viscosity of the fluid.

In this model The fluid satisfies the usual no-slip condition at the boundaries, while the dust particles may slip on the boundaries. However both the fluid and dust particles are subject to initial condition. Now see pg. 9)

The field equations (1.11) and (1.12) together with the continuity equations (1.2) and (1.8) form the back-bone of the work presented in the subsequent chapters II - V.

1.3.2 Neyfeh's Model

Saffman's model for dusty gases assumes that the bulk concentration of dust particles is small enough to be neglected. On the other hand the density of the material of dust is assumed to be fairly large compared to gas density so that the mass concentration of dust is an appreciable fraction of unity. This assumption is often justified, but at high fluid densities or at high particle mass-fraction, the volume-fraction of dust particles may become significantly large in which case it cannot be neglected. Rudinger (1965) has shown that in a flow analysis of gas-particle mixtures the errors involved by neglecting the volume-fraction of dust particles vary from insignificant to large. Hence Nayfeh (1966) has constructed a linear model for two-phase flow taking the volume-fraction of dust particles into account.

If ϕ denotes the volume occupied by the dust particles per unit volume of the mixture, and assuming the fluid flow relative to the particles to be a Stokes flow, the equations of conservation of momentum for the fluid and dust particles as formulated by Nayfeh (1966) are

$$\rho(1-\phi)\left[\frac{\partial \bar{u}}{\partial t} + (\bar{u}\,.\,\nabla)\bar{u}\right] = (1-\phi)\left[-\nabla p + \mu\nabla^2\bar{u}\right] + KN(\bar{v}-\bar{u}) \qquad (1.13)$$

$$mN\left[\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\bar{v}\right] = \phi\left[-\nabla p + \mu \nabla^2 \bar{u}\right] + KN(\bar{u} - \bar{v}) \quad \ldots \quad (1.14)$$

where the symbols used have the same meanings as those given in equations (1.1) and (1.5). The equations of continuity for the two phases are given by (1.2) and (1.8) assuming incompressibility of fluid and constant number density of dust particles.

In this model the presence of dust particles is characterized by the three parameters:

i) the mass concentration $\lambda = mN/\rho$,

ii) the volume-fraction $\phi = mN/\rho_1$ (ρ_1 being the density of the material comprising the dust particles) and

iii) the relaxation time $\tau = m/K$.

Using these parameters equations (1.13) and (1.14) take the form:

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla)\bar{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \bar{u} + \frac{\lambda}{\tau(1-\phi)}(\bar{v} - \bar{u}) \quad \ldots \quad (1.15)$$

$$\tau\left[\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\bar{v}\right] = \frac{\phi\tau}{\lambda}\left(-\frac{1}{\rho}\nabla p + \nu\nabla^2\bar{u}\right) + \bar{u} - \bar{v} \quad \ldots \quad (1.16)$$

When the volume-fraction ϕ is neglected i.e. putting $\phi = 0$ in (1.15) and (1.16), equations (1.11) and (1.12) of Saffman's model are recovered.

As in Saffman's model, the fluid particles are restricted by the usual no-slip boundary condition and initial condition, while dust particles satisfy only initial condition.

Equations (1.15) and (1.16) form the basic equations for the problem discussed in Chapter VI.

1.3.3 Drew's Model

Several macroscopic or continuum models have been proposed and used in the literature. All such models seem to differ in various details. Drew (1975) has constructed a simple continuum linear model for two-phase flow by considering the particulate phase as a continuum and using a drag force which is linear in the velocity difference. He has included the effect of the fluid pressure on the particles in the manner shown by Drew and Segel (1971), for in some situations the pressure force on the particles has been shown to be important (Drew (1974)). Further, the volume-fraction of the particles is assumed to be a constant. This assumption is exactly true in the limit of low concentrations. Thus the two sets of Navier-Stokes equations for the two phases as formulated by Drew (1975) are

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla)\bar{u} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \bar{u} + M'(\bar{v} - \bar{u}) \qquad (1.17)$$

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla)\bar{v} = -\frac{1}{\rho_p} \nabla P_p + D \nabla^2 \bar{v} + M(\bar{u} - \bar{v}) \qquad (1.18)$$

In these equations $\bar{u}$, $\bar{v}$ are the fluid and dust velocities, P is the pressure in the fluid, P_p is the partial pressure due to the particulate phase, $\rho = \bar{\rho}(1 - \phi)$ and $\rho_p = \bar{\rho}_p \phi$ are the densities of the fluid and dust phases respectively with $\bar{\rho}$ and $\bar{\rho}_p$ as the material densities and ϕ is the volume-fraction of the particulate phase, ν is the kinematic viscosity of the fluid, D is the diffusivity constant for the particulate phase, $M = 1/\tau$ where τ is the relaxation time of the dust particles and $M' = \rho_p M/\rho$.

Assuming the fluid as incompressible and the number density of dust particles to be a constant throughout the motion, the equations of conservation of mass for the fluid and dust continuums are given by (1.2) and (1.8).

In this model the fluid and dust phases each satisfy the no-slip boundary condition and initial condition. Equations (1.17) and (1.18) form the basis for the work presented in Chapter VII.

The field equations for the fluid and dust phases in the three models mentioned above are coupled Navier-Stokes equations. Since the Navier-Stokes equations are non-linear exact solutions are not possible in general, even in the classical single phase motion, except in a very few special cases. Thus, even though Saffman's model is relatively simple, exact solutions can be found only in a very limited number of cases. In order to solve problems of two-phase flows, it often becomes necessary to make certain assumptions such as taking the number density of dust particles to be constant and the mass concentration of dust to be small and/or uniform in space/time.

1.4 Present Investigations

The next nine chapters deal with some theoretical investigations on fluid flows with and without particulate suspensions. They are presented in two parts. Part I consists of six chapters relating to incompressible, viscous, dusty fluid flows. Chapters II to V employ Saffman's model, chapter VI uses Nayfeh's model and chapter VII is based on Drew's model. Part II consists of three chapters dealing with incompressible, viscous fluid flows without dust particles embedded in them.

Chapter II comprises the study of two problems dealing with the quasi-steady flow of an incompressible, viscous, dusty fluid between two parallel plates oscillating in their own planes with same frequency, but with difference in phase and amplitude. In the first problem, an oscillatory body force is applied parallel to the plates, having the same frequency as that of the plates, but with a phase different from that of either of the two plates. The amplitude of the body force is assumed to be an arbitrary function of the space coordinate. Some interesting cases of the body force are discussed. In the second problem, the body force is replaced by a uniform transverse magnetic field and the induced field is neglected. Exact solutions for the velocities of fluid and dust particles are obtained, and the influence of dust parameters, geometric parameters and the magnetic field on the flow characteristics are investigated. It is found that the flow is retarded by the presence of dust particles.

Chapter III investigates the unsteady, adjacent, laminar flow of two immiscible, incompressible, viscous, dusty fluids between two parallel plates driven by a constant pressure gradient by employing explicit finite difference technique. It is found that the velocities of dust particles are much less than that of dusty fluids. Further this problem is reconsidered using the Crank-Nicolson implicit finite difference scheme for the case when the two immiscible fluids are dust-free and the solutions thus obtained are compared with those got by the explicit method.

Chapter IV deals with two problems relating to the flow of a dusty, viscous fluid between two infinite, co-axial, circular cylinders one of which is kept stationary and the other is impulsively brought to rest from a state of uniform motion. In the first problem, the axially moving cylinder is impulsively stopped, while in the second problem, the uniformly rotating cylinder is brought to a sudden rest. Exact solutions are obtained by using the techniques of Laplace transform and the finite Hankel transform. The variation of flow rate, skin-friction and torque on the cylinders are discussed for a spectrum of values of dust parameters. Two interesting limiting cases are deduced.

In chapter V, a study is made of the quasi-steady azimuthal flow of a dusty fluid between two coaxial cylinders in oscillatory rotation, having the same frequency of oscillation but with different phase and amplitude. Fluctuating flows inside and outside a cylinder in oscillatory rotation are deduced as limiting cases. For small frequencies of excitation, the velocities of fluid and dust particles are equal. For large frequencies, the dust particles are stationary and the fluid flow has a boundary layer character.

Chapter VI analyses the general, unsteady, laminar flow of a dusty fluid between two parallel plates taking the volume-fraction of the dust particles into account. The flow is produced by the boundaries, with the lower plate moving parallel to itself with a time-dependent velocity, while the motion of the upper plate in its own plane is such that a linear combination of its velocity and shear stress is an arbitrary function of time, Using Laplace transform technique exact solutions are obtained for the fluid and dust velocities. Two interesting classes of physical problems are deduced. It is observed that the volume-fraction parameter has an increasing effect on velocity distributions and other principal flow characters.

In chapter VII, the unsteady flow of a dilute suspension between two contracting/distending, rectangular/

circular plates is investigated, employing Drew's continuum model. Using similarity solution technique the two sets of Navier-Stokes equations for the two phases are reduced to two fourth order, non-linear, coupled, ordinary differential equations which are solved numerically. The effects of the Schmidt number and the Reynolds number on the flow characteristics are examined.

Chapter VIII is devoted to the study of the flow of two immiscible, incompressible, viscous fluids occupying the semi-infinite region over an infinite flat plate set into motion in its plane with velocity $u_0 t^n \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} (\omega t)$. Using Laplace transform, exact solutions are obtained for the particular cases when the plate performs simple harmonic oscillations and when the plate is impulsively started from rest with constant velocity and constant acceleration. In each case the wall shear stress is computed for a range of values of density and viscosity ratios of the two fluids.

In chapter IX, an analysis is presented of the unsteady flow of an incompressible, viscous fluid between two parallel, infinitely long rectangular plates and circular plates, when the lower plate is fixed and the upper plate moves towards the lower plate. Full Navier-Stokes equations are used to obtain the pressure distribution as a function of the film thickness and the velocity of the upper plate. The sinkage relation between film thickness and time is determined for a given load on the upper plate. The departure from the classical inertialess solution is exhibited for various values of two dimensionless parameters involved, one characterizing the load and the other gravity.

The final chapter X reconsiders the problem of chapter IX when the lower plate is a stretching sheet. Eliminating pressure from the Navier-Stokes equations a fourth order, non-linear, ordinary differential equation is obtained and this is solved numerically. Velocity distributions and pressure variation across the plates are determined.