STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011–02 & thereafter)

SUBJECT CODE : 11MT/RC/AA105

M.Phil. DEGREE EXAMINATION, JANUARY 2014
MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ALGEBRA AND ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

Answer any five questions. Each question carries 20 marks:

1. (a) Prove that a bijective map of a lattice L into a lattice L' is a lattice isomorphism if and only if its inverse are order preserving.
   (b) State and prove the fundamental theorem of projective geometry. (5+15)

2. (a) Define a Noetherian and an Artinian module and give an example of each.
   (b) If R is a Noetherian ring, prove that the polynomial ring $R[x]$ is also a Noetherian ring. (6 + 14)

3. (a) Let R be a ring, and $M_n(R)$ be the ring of n x n matrices with entries in R. prove that categories mod-R and mod- $M_n(R)$ of right modules over R and $M_n(R)$ respectively are equivalent.
   (b) Define tensor product of a right R – module M and a left R – module N and prove that it exists.
   (c) Prove the following: (i). $Q \otimes_Z Z_8 = 0$. (ii). $Z_6 \otimes_Z Z_7 = 0$. (8 + 8 + 4)

4. (a) State and prove Jordan – Holder- Dedekind theorem on lattices.
   (b) State and prove Riesz Representation theorem. (10 + 10)

5. (a) State and prove the Lebesgue's monotone convergence theorem.
   (b) State and prove the Lebesgue's dominated convergence theorem. (10 +10)

6. (a) State and prove Holder's inequality and Minkowski's inequality.
   (b) Prove that $L_p(\mu)$ is a complete metric space, for $1 \le p \le \infty$ for every positive measure μ. (10 +10)

7. State and prove Plancheral's theorem. (20)

8. (a) For an R-module M, prove that the following conditions are equivalent.
   (i) M is Noetherian
   (ii) Every submodule of M is finitely generated.
   (iii) Every non-empty set S of submodules of M has a maximal element.
   (b) If R is Noetherian ring, prove that each ideal contains of R contains a finite product of prime ideals of R. (14 +6)

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