STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2011–02 & thereafter)

SUBJECT CODE : 11MT/RC/AA105

M.Phil. DEGREE EXAMINATION, JANUARY 2013
MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ALGEBRA AND ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

Answer any five questions. Each question carries 20 marks:

1. (a) Prove that the lattice of normal subgroups of a group G is modular.
   (b) Prove that a lattice is modular if and only if whenever $a \geq b$ and $a \wedge c = b \wedge c$ and $a \vee c = b \vee c$ for some c in L, then $a = b$.
   (c) Define a Boolean algebra and prove that the complement of any element of a Boolean algebra is unique. (5+10+5)

2. (a) If a module M contains a submodule N such that N and M/N are Artinian, then prove that M is Artinian.
   (b) Let R be a left Artinian ring with unity and no non-zero nilpotent ideals. Then prove that R is isomorphic to a finite direct sum of matrix rings over division rings. (5 +15)

3. (a) Define tensor product of modules and prove that it exists and unique.
   (b) Prove the following:
   (i) $Q \otimes_Z Z_8 = 0$ (ii) $Z_6 \otimes_Z Z_7 = 0$ (iii) $Q \otimes_Z Z = Q$ as additive groups.
   (14 + 6)

4. State and prove the fundamental theorem of projective geometry. (20)

5. (a) State and prove Lebesgue monotone convergence theorem.
   (b) State and prove Fatou's Lemma. (15+5)

6. (a) Prove that $L^p(\mu)$ is a complete metric space, for $1 \leq p \leq \infty$ and for every positive measure μ.
   (b) If $f \in L^1$, prove that $\hat{f} \in C_0$ and $\|\hat{f}\|_\infty \leq \|f\|_1$. (15+5)

7. State and prove Plancherel theorem. (20)

8. (a) Define a uniform module and give an example.
   (b) Prove that the ring $R = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} | a \in Z, \ b, c \in Q\}$ is not left Noetherian but it is right Noetherian.
   (c) Is Z considered as a module over itself an Artinian (Noetherian) module? Justify your answer. (4+10+6)

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