STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09 & thereafter)

SUBJECT CODE : MT/RC/AM105

M.Phil. DEGREE EXAMINATION, JANUARY 2011
MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : METHODS OF APPLIED MATHEMATICS
TIME : 3 HOURS **MAX. MARKS : 100**

Answer four questions without omitting any section. Each question carries 25 marks.

SECTION – A

1. a) State the sufficient conditions for the existence of Laplace transform of a function. Show that these conditions are not necessary by giving an appropriate example.

   b) Find the Laplace transform of $\frac{\sin at}{t}$. Does the Laplace transform of $\frac{\cos at}{t}$ exist?

   c) Using convolution theorem, find $L^{-1}\left[\frac{1}{(1+p)^2 p^2}\right]$.

2. a) Solve: $\frac{\partial y}{\partial x} = y + 2\frac{\partial y}{\partial t}$, $y(x,0) = 6e^{-3x}$, which is bounded for $x > 0, t > 0$.

   b) A semi-infinite solid $x > 0$ is initially at temperature zero. At time $t = 0$ a constant temperature $u_0 > 0$ is applied and maintained at the face $x = 0$. Find the temperature at any point of the solid at any later time $t > 0$.

3. a) Find the Fourier transform of $f(x)$ given by $f(x) = 1$, $|x| < a$ and $f(x) = 0, |x| > a$ and hence evaluate $\int_0^\infty \frac{\sin pa \cos px}{p} dp$.

   b) Find Fourier sine and cosine transform of x^{n-1}.

   c) Solve the integral equation : $\int_0^\infty f(x)\cos px\,dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$.

4. a) Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 4$, $t > 0$, subject to the conditions $u(0,t) = 0$, $u(4,t) = 0$ and $u(x,0) = 2x$. Give a physical interpretation of the problem.

   b) Use Parseval's identity for Fourier transform to evaluate
   $\int_0^\infty \frac{\sin at}{t(t^2 + a^2)} dt$.

SECTION – B

5. a) Find the curve joining given points A and B which is traversed by a particle moving under gravity from A to B in the shortest time (ignore friction along the curve and resistance of the medium).

b) Find an approximate solution of the Poisson's equation

$I(z)=\iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2-2z\right]dxdy$ where D is the domain

$-a\le x\le a,\ -b\le x\le b$ and $z=0$ on the boundary of D.

6. a) Using Raylegh-Ritz method, find an approximate solution to the extremum of the functional $I[y(x)]=\int_0^1\left(y'^2+y^2\right)dx,\quad y(0)=0,\ y(1)=1$.

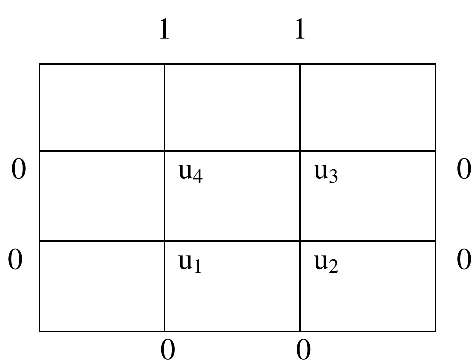
b) Determine the extremum of the functional

$$I[y(x)]=\int_{-l}^{l}\left(\frac{1}{2}\mu y''^2+\rho y\right)dx,\ y(-l)=0,\ y(l)=0,\ y'(-l)=0,\ y'(l)=0.$$

7. Explain the Galerkin method and use this method to solve $y''+y=3x^2,\ y(0)=y(2)=3.5$

8. a) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x,0)=0,\ 0\le x\le 1;\ u(0,t)=0, u(1,t)=t$ using Crank-Nicolson method, taking $h=\frac{1}{4}, k=\frac{1}{16}$ (Do two time steps).

b) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ over the square mesh given below by Jacobi's method:



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