STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86.
(Effective from the academic year 2005- 2006)

M.Phil Degree Examination
April - 2011

Title: Functional Analysis
Code: MT/RO/FA25

Maximum Marks: 100
Total Time: 3 Hrs

Answer any five questions (5×20=100)

1. State and prove the Banach contraction principle.

2. State and prove the Hausdorff theorem on compactness.

3. a) Let $T \in B(X,X)$, where X is a Banach space. If $\|T\| < 1$, then prove that $(I-T)^{-1}$ exists as a bounded linear operator on the whole space X and
$(I-T)^{-1} = \sum_{j=0}^{\infty} T^j = I + T + T^2 + \cdots$

   b) Prove that the resolvent set $\rho(T)$ of a bounded linear operator T on a complex Banach space X is open; hence the spectrum is closed.

4. a) Let $T: H \to H$ be a bounded self adjoint linear operator on a complex Hilbert space H. Then prove that a number λ belongs to the resolvent set $\rho(T)$ of T if and only if there exists a $c > 0$ such that for every $x \in H$, $\|T_\lambda x\| \geq c\|x\|$.

   b) If two bounded self adjoint operators S and T on a Hilbert space H are positive and commute then prove that their product ST is positive.

5. In a complete space E, let $x(t)$ be a continuous function with $t \in [a,b]$ and let A_{τ_n} be a sequence of partitions of the interval $[0,1]$ such that $\tau_n \to 0$ as $n \to \infty$. If we form the integral sums $S[A_{\tau_n}, x(t)]$ then prove that they tend to a limit $S = \lim_{n\to\infty} S[A_{\tau_n}, x(t)]$.

6. Prove that if the weak differential exists in a neighbourhood then Frechet differential exists and they are equal in that neighbourhood.

7. Define extremal points and Haar condition. And prove that Haar condition is sufficient for the uniqueness of the best approximations.

8. State and prove that Haar uniqueness theorem on best approximation.