STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86.

(For candidates admitted in the year 2009 -2010 and there after)

DEPARTMENTOF MATHEMATICS

SUBJECT CODE: MT/RO/AA205

M Phil DEGREE SEMESTER EXAMINATION APRIL – 2011

CORSE : OPTIONAL TIME:3 HOURS

PAPER : ADVANCE ALGEBRA MAX MARKS: 100

ANSWER ANY FIVE **(5 × 20 = 100)**

1. Define semi simple R- modules with an example. State and prove Jacobson's density theorem on semi simple R- modules

2. a) Prove the Structure theorem for semi simple Rings.
   b) State and prove Morita theorem on R – module.

3. a) Prove that every sub-module and every factor module of a semi simple module is semi- simple.
   b) Prove that for fixed R-modules and K= End $_X(En)$ = Mat $_n(K)$.

4. State and prove Wedderburn's Structure theorem on semi-simple R – algebras

5. a) Prove that if A is a simple Algebra , then every homomorphic image of A is semi-simple.

   b) If A is an R- algebra and M is a free right A- module on generators then prove that

   $$E_A(M) \simeq M_n(E_A(M))$$

6. a) Prove that every Z- module can be embedded in an injective Z- module
   b) Define projective and injective in modules and prove that the Z – module Q is injective

7. a) State the Five Lemma theorem on Exact sequence..
   b) State and prove Schannuel's lemma for projective module.

8. a) Define the following
      i. Sub direct product
      ii. Lattice of congruence
      iii. Band congruence and decomposition
      iv. Free semi groups
      v. Completely prime Ideals and filters

   b) Prove that every finitely generated projective A – module P is finitely presented.