STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2009-10 & thereafter)

SUBJECT CODE : MT/PC/MI24

M. Sc. DEGREE EXAMINATION, APRIL 2011
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : MEASURE THEORY AND INTEGRATION
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

1. If $m^*(E) = 0$, then prove that E is measurable.
2. Show that for any set A and any $\varepsilon > 0$, there exists an open set O such that A is contained in O & $m^*(O) \leq m^*(A) + \varepsilon$.
3. State and prove Labesgue Dominated Convergence Theorem .
4. Prove that the measurability of f is equivalent to the following:
   i) for all α ,$\{ f(x) \geq \alpha\} \in \mathbb{S}$,
   ii) for all α ,$\{x:f(x) < \alpha\} \in \mathbb{S}$,
   iii) for all α , $\{x:f(x) \leq \alpha\} \in \mathbb{S}$, where $\mathbb{S}$ is a σ – algebra.
5. Prove that a countable union of sets positive with respect to a signed measure v is a positive set.
6. Let μ be a signed measure on $[[X, \mathbb{S}]]$ and let v be a finite-value signed measure on $[[X, \mathbb{S}]]$ such that $v << \mu$. Then prove that given $\varepsilon >0$ there exists $\delta >0$ such that $|v|(E) < \varepsilon$ Whenever $|\mu|(E) < \delta$.
7. Prove that the class of elementary sets E is an algebra.

SECTION – B

ANSWER ANY THREE QUESTIONS: **(3 X20 = 60)**

8. Suppose that f is any extended real-valued function which for every x and y satisfies $f(x) + f(y) = f(x + y)$
   i) Show that f is either everywhere finite or everywhere infinite.
   ii) Show that if f is measurable and finite, then $f(x) = xf(1)$.

9. State and prove Fatou's Lemma.

10. If μ is a measure on a σ- ring $\mathbb{S}$, then prove that the class $\overline{\mathbb{S}}$ of sets of the form $E \Delta N$ for any sets E,N such that $E \in \mathbb{S}$ while N is contained in some set in $\mathbb{S}$ of zero Measure, is a σ- ring, and the set function $\bar{\mu}$ defined by $\bar{\mu}(E \Delta N) = \mu(E)$ is a complete measure on $\overline{\mathbb{S}}$.

11. Let v be a signed measure on $[[X , \mathbb{S}]]$. Let $E \in S$ and $v(E) > 0$. Then prove that there exists A , a set positive with respect to v , such that A is contained in E , and $v(A)>0$.

12. Let $[[X, \mathbb{S},\mu]]$ and $[[Y,\mathfrak{T},v]]$ be σ-finite measure spaces. For $V \in \mathbb{S} \times \mathfrak{T}$ write $\phi(x)=v(V_x)$ & $\psi(y)=\mu(V^y)$,for each $x \in X$, $y \in Y$. prove that ϕ is $\mathbb{S}-$ measurable , ψ is $\mathfrak{T}$ -measurable and $\int_X \phi \, d\mu = \int_Y \psi \, dv$.