STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2009–10)

SUBJECT CODE: MT/PC/FA44

M. Sc. DEGREE EXAMINATION, APRIL 2011
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : FUNCTIONAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

1. If N is a normed linear space and x_0 is a non-zero vector in N, then prove that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
2. If M is a closed linear subspace of a Hilbert Space H, then prove that $H = M \oplus M^{\perp}$.
3. State and prove Bessel's inequality.
4. Prove that every Hilbert Space is reflexive.
5. Prove that two matrices in A_n are similar if and only if they are the matrices of a single operator on H relative to different bases.
6. Prove: If $P_1, P_2, \dots, P_n$ are the projections on closed linear subspaces $M_1, M_2, \dots, M_n$ of H, then $P = P_1, P_2, \dots, P_n$ is a projection $\Leftrightarrow$ the P_i's are pairwise orthogonal and P is the projection on $M = M_1, M_2, \dots, M_n$.
7. With usual notations show that Z is a subset of S and the boundary of S is a subset of Z.

SECTION – B

ANSWER ANY THREE QUESTIONS: **(3 X 20 = 60)**

8. a) State and prove Minkowski's inequality.

   b) Let M be a linear subspace of a normed linear space N and let f be a functional defined on M. Let $x_0 \notin M$ and $M_0 = M + [x_0]$. Prove that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

9. a) State and prove parallelogram law in a Hilbert space.
   b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

10. a) If T is an operator on a Hilbert Space H for which $(Tx, x) = 0$ for all $x \in H$, then prove that $T = 0$.
    b) If T is an operator on a Hilbert Space H, then prove that T is normal if and only if its real and imaginary parts commute.

11. State and prove the Spectral Theorem.

12. a) Prove that $\sigma(x)$ is a non empty compact subset of the complex plane.
    b) Prove $r(x) = \lim \|x_n\|^{\frac{1}{n}}$, x in a general Banach Algebra A.