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PERFECT INTUITIONISTIC FUZZY GRAPHS

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ABSTRACT

In this paper, μ-perfect, υ-perfect, complete μ-perfect and complete υ-perfect intuitionistic fuzzy graphs are defined. The radius, diameter, status, median and connectivity of perfect intuitionistic fuzzy graphs are discussed.

Keywords: Perfect intuitionistic Fuzzy graph, Distance, Eccentricity, Status and median.

1. INTRODUCTION

The basic idea of a fuzzy relation was defined by Zadeh [10]. Rosenfeld [9] considered fuzzy relations on fuzzy sets and developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya [2] obtained other graph theoretic results concerning center and eccentricity. A.Nagoor Gani and M. Basheer Ahamed [4] defined perfect fuzzy graph and complete perfect fuzzy graph and discussed some of its properties.

K.T. Atanassov [1] introduced the concept of Intuitionistic Fuzzy Graph (IFG) in 1994. Research on the theory of intuitionistic fuzzy sets has a vast opening in various applications. R.Parvathy and M.G.Karunambigai [8] gave a new definition for IFG and analyzed its components. A. Nagoor Gani and S. Shajitha Begum [7] defined status and median in IFGs. In this paper, we introduce the notion of μ-perfect, υ-perfect, complete μ-perfect and complete υ-perfect intuitionistic fuzzy graphs and some of its properties are analysed.

2. PRELIMINARIES

2.1. Definition: [8]
An Intuitionistic fuzzy graph is of the form G = < V, E > where

i) $V=\{v_1,v_2,\dots,v_n\}$ such that $\mu_1: V \to [0,1]$ and $\upsilon_1: V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i) + \upsilon_1(v_i) \le 1$ for every $v_i \in V$, ($i = 1,2, \dots n$),

ii) $E \subseteq V \times V$ where $\mu_2: V\times V \to [0,1]$ and $\upsilon_2: V\times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$, $\upsilon_2(v_i, v_j) \le \max[\upsilon_1(v_i), \upsilon_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \upsilon_2(v_i,v_j) \le 1$ for every $(v_i\, v_j) \in E$, ($i, j = 1,2, \dots n$) .

2.2. Definition[8]:An IFG G =<V,E> is a complete IFG, if μ_{2ij} = min (μ_{1i}, μ_{1j}) and υ_{2ij} = max (υ_{1i} , υ_{1j}) for all $v_i \in V$.

2.3. Definition[6]: Let G =<V,E> be an IFG. Then the order of G is defined to be O(G)= ($O_\mu(G)$,$O_\upsilon(G)$) where $O_\mu(G) = \Sigma_{v\in V}\ \mu_1(v)$ and $O_\upsilon(G) = \Sigma_{v\in V}\ \upsilon_1(v)$.

2.4. Definition[6]: The size of G is defined to be S(G)= ($S_\mu(G)$, $S_\upsilon(G)$) where $S_\mu(G) = \Sigma_{u\neq v}\ \mu_2(u,v)$ and $S_\upsilon(G) = \Sigma_{u\neq v}\ \upsilon_2(u,v)$.

2.5. Definition[6]: Let G = < V, E > be an IFG. The neighbourhood of any vertex v is defined as N(v)=($N_\mu(v)$,$N_\upsilon(v)$) where $N_\mu(v)=\{w\in V;\ \mu_2(v, w) = \mu_1(v) \wedge \mu_1(w)\}$and $N_\upsilon(v)=\{w\in V;\ \upsilon_2(v,w)= \upsilon_1(v) \vee \upsilon_1(w)\}$ and N[v] = N(v) U {v} is called the closed neighbourhood of v.

2.6. Definition[7]:The μ–distance $\delta_\mu(v, w)$ is defined to be the minimum of the μ-lengths of all the paths joining v and w. (i.e.) $\delta_\mu(v, w) = \wedge \{L_\mu(P)$: P is a path between v and w}.

The υ–distance $\delta_\upsilon(v, w)$ is defined to be the maximum of the υ-lengths of all the paths joining v and w. (i.e.) $\delta_\upsilon(v, w) = \vee \{L_\upsilon(P)$: P is a path between v and w}.

2.7. Definition[7]: Let G = < V, E > be an IFG. The eccentricity of a node v is defined as e(v) = ($e_\mu(v)$,$e_\upsilon(v)$) where the μ–eccentricity $e_\mu(v)$ is the maximum of all the μ-distances $\delta_\mu(v, w)$ for every w є V. (i.e.) $e_\mu(v) = \vee \{ \delta_\mu(v, w), w \in V \}$ and the γ-eccentricity $e_\upsilon(v)$ is the maximum of all the υ-distances $\delta_\upsilon(v, w)$ for every w є V. (i.e.) $e_\upsilon(v) = \vee \{ \delta_\upsilon(v, w), w \in V \}$.

2.8. Definition[7]: A radius of an IFG is r(G) = ($r_\mu(G)$,$r_\upsilon(G)$) where the μ–radius $r_\mu(G)$ is the minimum of all the μ–eccentrities of the vertices of G and υ–radius $r_\upsilon(G)$ is the minimum of all the υ–eccentricities of the vertices of G.

2.9. Definition[7]: The diameter d(G) = ($d_\mu(G)$,$d_\upsilon(G)$) where μ-diameter $d_\mu(G)$ is the maximum of all the μ–eccentricities of the vertices of G and $d_\upsilon(G)$ is the maximum of all the υ–eccentricities of the vertices of G.

2.10. Definition[7]**:**The μ-status $s_\mu(v)$ of G is defined to be the sum of all the μ-distances $\delta_\mu(v, w)$ for every w є V. i.e. $s_\mu(v) = \Sigma\ \delta_\mu(v, w)$, w є V

The υ-status $s_\upsilon(v)$ of G is defined to be the sum of all the υ-distances $\delta_\upsilon(v, w)$ for every w є V. i.e. $s_\upsilon(v) = \Sigma\ \delta_\upsilon(v, w)$, w є V

2.11. Definition[7]**:**The Median is defined as M(G) = ($M_\mu(G)$,$M_\upsilon(G)$) where $M_\mu(G)$ is the set of nodes with minimum μ status and $M_\upsilon(G)$ is the set of nodes with minimum υ status.

2.12. Definition[6]**:** The closed neighbourhood degree of a vertex 'v' is defined as $d_N[v]=(d_{N\mu}[v],d_{N\upsilon}[v])$ where $d_{N\mu}[v] = \Sigma_{w\ \varepsilon\ N(v)}\ \mu_1(w) + \mu_1(v)$ and $d_{N\upsilon}[v] = \Sigma_{w\ \varepsilon\ N(v)}\ \upsilon_1(w) + \upsilon_1(v)$.

2.13. Definition[6]**:** An intuitionistic fuzzy graph G = < V, E > is said to be regular, if all the vertices have the same closed neighbourhood degree.

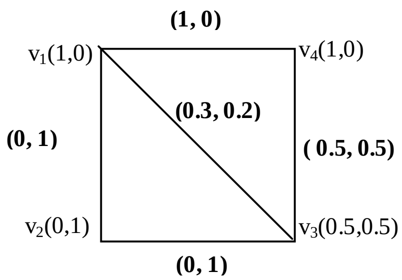
2.14 Definition[5]**:** If $v_i, v_j \in V \subseteq G$, then the μ-strength of connectedness between v_i and v_j is $CONNG_\mu(v_i, v_j)$ = sup { μ_2^k (v_i, v_j) / k = 1,2,...,n) } and υ-strength of connectedness between v_i and v_j is $CONNG_\upsilon(v_i, v_j)$ = inf { υ_2^k (v_i, v_j) / k = 1,2,...,n)}.

3. PERFECT INTUITIONISTIC FUZZY GRAPHS

3.1. Definition: Let G = < V, E > be an IFG. A vertex is called a μ_1-perfect intuitionistic fuzzy vertex if μ_1 (v) = 1 and a υ_1-perfect intuitionistic fuzzy vertex if υ_1(v) = 1 for some v∈V.

An edge (v, w) is called a μ_1-perfect intuitionistic fuzzy edge if μ_2(v, w) = 1 and a υ_1-perfect intuitionistic fuzzy edge if υ_2 (v, w) = 1 for some (v, w) ∈ E.

3.2 Example:



v_1, v_4	- μ_1 - perfect intuitionistic fuzzy vertex
v_2	- υ_1 - perfect intuitionistic fuzzy vertex
(v_1, v_4)	- μ_2 - perfect intuitionistic fuzzy edge
(v_1, v_2), (v_2, v_3)	- υ_2 - perfect intuitionistic fuzzy edge

Fig. 1. Intuitionistic fuzzy graph

3.3 Definition: An IFG G = < V, E > is called a μ_1-perfect intuitionistic fuzzy graph if μ_1 (v) = 1for all v∈V and a υ_1-perfect intuitionistic fuzzy graph if υ_1(v) = 1 for all v∈ V.

3.4 Example:

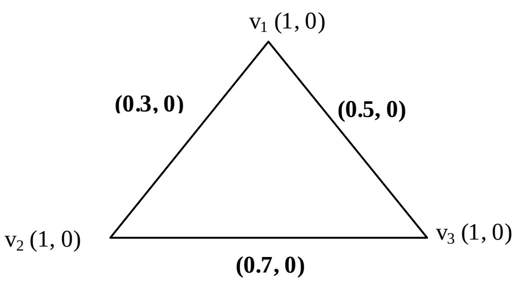


Fig. 2. μ_1 - perfect IFG

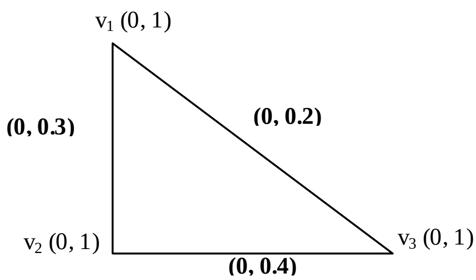


Fig. 3. υ_1 - perfect IFG

3.5 Definition: An IFG G = < V, E > is called a μ_2-perfect intuitionistic fuzzy graph if μ_2 (v, w) = 1 for all (v, w) ∈E and υ_2 - perfect intuitionistic fuzzy graph if υ_2 (v, w) = 1for all (v, w)∈ E.

3.6 Example:

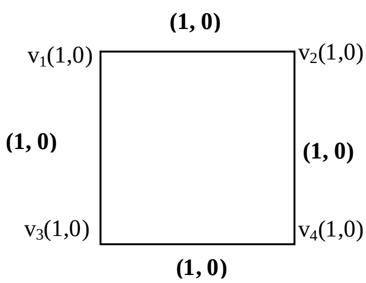


Fig. 4. μ_2 - perfect IFG

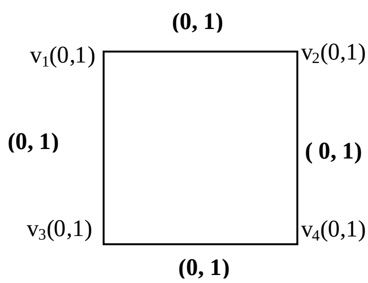


Fig. 5. υ_2 - perfect IFG

3.7 Remark:

i. Every μ_2-perfect intuitionistic fuzzy graph is μ_1-perfect intuitionistic fuzzy graph, converse need not be true.

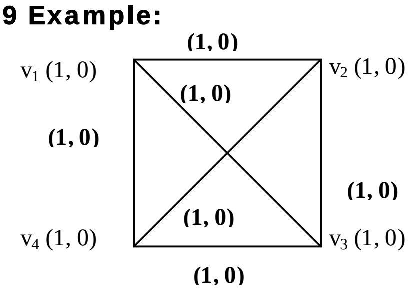
For example, the graph given in Fig.4 is μ_2-perfect intuitionistic fuzzy graph and μ_1-perfect intuitionistic fuzzy graph whereas the graph given in Fig.2 is μ_1-perfect intuitionistic fuzzy graph but not μ_2-perfect intuitionistic fuzzy graph.

ii. Every υ_2-perfect intuitionistic fuzzy graph is υ_1-perfect intuitionistic fuzzy graph, converse need not be true.

For example, the graph given in Fig.5 is υ_2-perfect intuitionistic fuzzy graph and υ_1-perfect intuitionistic fuzzy graph whereas the graph given in Fig.3 is υ_1-perfect intuitionistic fuzzy graph but not υ_2-perfect intuitionistic fuzzy graph.

3.8 Definition: An IFG G = < V, E > is called a complete μ_2-perfect intuitionistic fuzzy graph if μ_2 (v, w) = 1for all (v, w) $\in$V and a complete υ_2 - perfect intuitionistic fuzzy graph if υ_2 (v, w) = 1for all (v, w)$\in$ V.

3.9 Example:



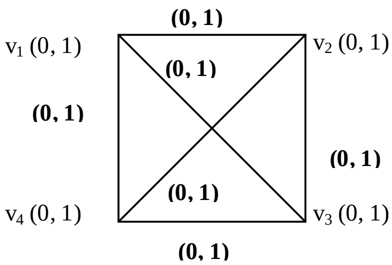


Fig. 6. Complete μ_2 - perfect IFG **Fig. 7. Complete υ_2 - perfect IFG**

3.10 Remark:

i. Every complete μ_2-perfect intuitionistic fuzzy graph is μ_2-perfect intuitionistic fuzzy graph, converse need not be true.

For example, the graph given in Fig.6 is complete μ_2-perfect intuitionistic fuzzy graph and μ_2-perfect intuitionistic fuzzy graph whereas the graph given in Fig.4 is μ_2-perfect intuitionistic fuzzy graph but not complete μ_2-perfect intuitionistic fuzzy graph.

ii. Every complete υ_2-perfect intuitionistic fuzzy graph is υ_2-perfect intuitionistic fuzzy graph, converse need not be true.

For example, the graph given in Fig.7 is complete υ_2-perfect intuitionistic fuzzy graph and υ_2-perfect intuitionistic fuzzy graph whereas the graph given in Fig.5 is υ_2-perfect intuitionistic fuzzy graph but not complete υ_2-perfect intuitionistic fuzzy graph.

3.11 Definition: An IFG G = < V, E > is called a μ-perfect intuitionistic fuzzy graph if it has a μ_2-perfect intuitionistic fuzzy graph and a υ-perfect intuitionistic fuzzy graph if it has a υ_2-perfect intuitionistic fuzzy graph.

3.12 Definition: An IFG G = < V, E > is called complete μ-perfect intuitionistic fuzzy graph if it has a μ_2-perfect intuitionistic fuzzy graph and complete υ-perfect intuitionistic fuzzy graph if it has a υ_2-perfect intuitionistic fuzzy graph.

3.13 Remark:

Every complete μ-perfect intuitionistic fuzzy graph is a μ-perfect intuitionistic fuzzy graph, but not conversely true.

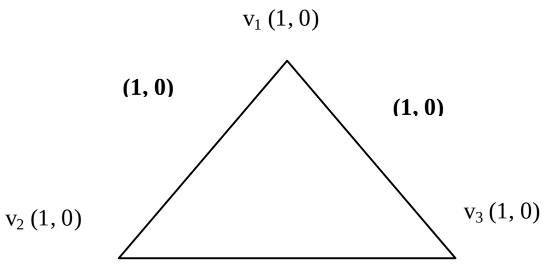


Fig. 8. : μ - perfect IFG and Complete μ - perfect IFG

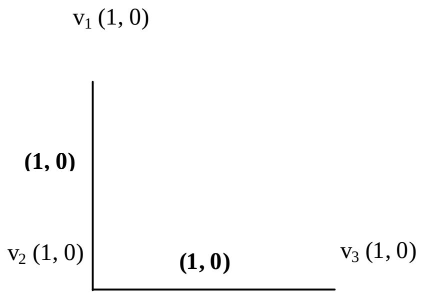


Fig. 9: μ - perfect IFG not Complete μ - perfect IFG

Every complete υ-perfect intuitionistic fuzzy graph is a υ-perfect intuitionistic fuzzy graph, but not conversely true.

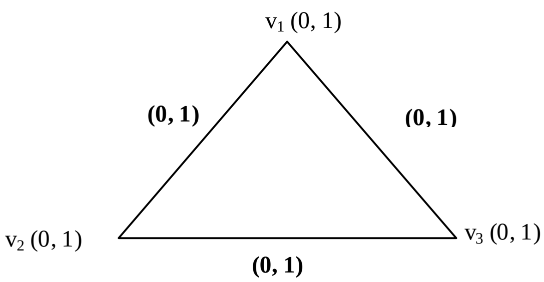


Fig. 10 : υ - perfect IFG and Complete υ - perfect IFG

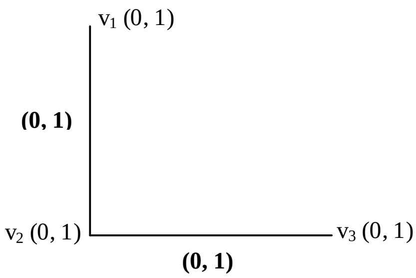


Fig. 11 : υ - perfect IFG not Complete υ - perfect IFG

4. STATUS IN PERFECT IFGS

4.1 Theorem: The radius of a complete μ-perfect intuitionistic fuzzy graph with n vertices is always (1, 0) and complete υ-perfect intuitionistic fuzzy graph with n vertices is always (0, n-1) .

Proof:

Let G = <V,E> be a complete μ-perfect intuitionistic fuzzy graph with n vertices. The eccentricity e(v) = $(e_\mu(v), e_\upsilon(v))$ of each vertex is always (1, 0). We know that the radius is minimum eccentricity among the vertices. Hence the radius of a complete μ-perfect intuitionistic fuzzy graph is always (1,0).

Let G = <V,E> be a complete υ-perfect intuitionistic fuzzy graph with n vertices. The eccentricity e(v) = $(e_\mu(v), e_\upsilon(v))$ of each vertex is always (0, n-1). We know that the radius is minimum eccentricity among the vertices. Hence the radius of a complete υ-perfect intuitionistic fuzzy graph is always (0, n-1).

4.2 Corollary: The diameter of a complete μ-perfect intuitionistic fuzzy graph with n vertices is (1, 0) and complete υ-perfect intuitionistic fuzzy graph with n vertices is (0, n-1).

4.3 Example:

Consider the μ-perfect intuitionistic fuzzy graph in Fig. 6.
$e_\mu(v_1) = e_\mu(v_2) = e_\mu(v_3) = e_\mu(v_4) = (1, 0)$. Thus $r_\mu(G) = (1,0)$ and $d_\mu(G) = (1, 0)$.
Consider the υ-perfect intuitionistic fuzzy graph in Fig. 7.
$e_\upsilon(v_1) = e_\upsilon(v_2) = e_\upsilon(v_3) = e_\upsilon(v_4) = (0, 3)$. Thus $r_\upsilon(G) = (0, 3)$ and $d_\upsilon(G) = (0, 3)$.

4.4 Theorem: Every complete μ-perfect intuitionistic fuzzy graph is self μ-centered and every complete υ-perfect intuitionistic fuzzy graph is self υ-centered.

Proof:

Let G = <V,E> be a complete μ-perfect intuitionistic fuzzy graph with n vertices. The eccentricity $e(v) = (e_\mu(v), e_\upsilon(v))$ of each vertex is always (1, 0). A connected IFG is self μ-centered if each node has same μ-eccentricity. Hence a complete μ-perfect intuitionistic fuzzy graph is self μ-centered.

Let G = <V,E> be a complete υ-perfect intuitionistic fuzzy graph with n vertices. The eccentricity $e(v) = (e_\mu(v), e_\upsilon(v))$ of each vertex is always (0, n-1). A connected IFG is self υ-centered if each node has same υ-eccentricity. Hence a complete υ-perfect intuitionistic fuzzy graph is self υ-centered.

4.5 Theorem: The status of complete μ-perfect intuitionistic fuzzy graph is (O(G) – 1, 0) and complete υ-perfect intuitionistic fuzzy graph is (0, (n-1)(O(G) – 1)) where n is the number of vertices.

Proof:

Let G be a complete μ-perfect intuitionistic fuzzy graph with n vertices. Then we have the distance $\delta(v, w) = (\delta_\mu(v, w), \delta_\upsilon(v, w)) = (1, 0)$ for any $v, w \in V$. The status of each vertex is sum of the distance between that vertex and all other vertices. Thus the status of each vertex is (O(G) – 1, 0).

Let G be a complete υ-perfect intuitionistic fuzzy graph with n vertices. Then we have the distance $\delta(v, w) = (\delta_\mu(v, w), \delta_\upsilon(v, w)) = (0, n-1)$ for any $v, w \in V$. The status of each vertex is sum of the distance between that vertex and all other vertices.

Thus the status of each vertex is (0, (n-1)(O(G) – 1)).

4.6 Corollary: The median of each vertex of a complete μ-perfect intuitionistic fuzzy graph is (O(G) – 1, 0) and a complete υ-perfect intuitionistic fuzzy graph is (0, (n-1)(O(G) – 1)) where n is the number of vertices.

4.7 Remark: In a complete μ-perfect intuitionistic fuzzy graph and complete υ-perfect intuitionistic fuzzy graph, all vertices are median vertices.

4.8. Example:
Consider the complete μ-perfect intuitionistic fuzzy graph in Fig.8. Here n = 3 and O(G) = (3, 0).
$s(v_1) = s(v_2) = s(v_3) = (2, 0)$. Hence s(G) = (2, 0).
Also the median M(G) = (2, 0) and Median vertex set = $\{ v_1, v_2, v_3 \}$.

Consider the complete υ-perfect intuitionistic fuzzy graph in Fig. 10. Here n = 3 and O(G) = (3, 0).

Here $s(v_1) = s(v_2) = s(v_3) = (0, 4)$. Hence s(G) = (0, 4).
Also the median M(G) = (0, 4) and Median vertex set = { v_1, v_2, v_3 }.

4.9 Theorem: The total status of a complete μ-perfect intuitionistic fuzzy graph is (n[O(G) – 1], 0) and a complete υ-perfect intuitionistic fuzzy graph is (0, n(n-1)(O(G) – 1)) where n is the number of vertices.

Proof:
Let G be a complete μ-perfect intuitionistic fuzzy graph with n vertices. By theorem 4.5, the status of each vertex of G is (O(G) – 1, 0). The total status of an IFG is $t[s(G)] = (t_\mu[s(G], t_\upsilon[s(G])$ where $t_\mu[s(G]$ is the sum of μ-status of all the vertices of G and $t_\upsilon[s(G]$ is the sum of υ-status of all the vertices of G. Hence the total status of a complete μ-perfect intuitionistic fuzzy graph is (n[O(G) – 1], 0).

Similarly, let G be a complete υ-perfect intuitionistic fuzzy graph with n vertices. By theorem 4.5, the status of each vertex of G is (0, (n-1)[O(G) – 1]). By the definition of total status, we get the total status of a complete υ-perfect intuitionistic fuzzy graph is (0, n(n-1)(O(G) – 1)).

4.10 Theorem: Every μ-perfect intuitionistic fuzzy graph and υ-perfect intuitionistic fuzzy graph is a self median intuitionistic fuzzy graph.

Proof:
Let G be a complete μ-perfect intuitionistic fuzzy graph with n vertices and n ≥ 2. By theorem 4.5, the status of each vertex of G is (O(G) – 1, 0). Hence minimum status m[s(G)] = maximum status M[s(G)] = (O(G) – 1, 0). Thus G is a self median intuitionistic fuzzy graph.

Let G be a complete υ-perfect intuitionistic fuzzy graph with n vertices and n ≥ 2. By theorem 4.5, the status of each vertex of G is (0, (n-1)(O(G) – 1)).

Hence m[s(G)]=M[s(G)] = (0, (n-1)(O(G) – 1)). Thus G is a self median intuitionistic fuzzy graph.

4.11 Theorem: Every μ-perfect intuitionistic fuzzy graph and υ-perfect intuitionistic fuzzy graph is a regular intuitionistic fuzzy graph.

Proof:
Let G be a complete μ-perfect intuitionistic fuzzy graph or υ-perfect intuitionistic fuzzy graph with n vertices. Then every vertex of G gets equal degree. Then the closed neighbourhood degree of every vertex is the same. So we have $\delta_N(G)=(\delta_{N\mu}(G),\delta_{N\upsilon}(G)) = (\Delta_{N\mu}(G),\Delta_{N\upsilon}(G)) = \Delta_N(G)$. Thus G is regular intuitionistic fuzzy graph.

4.12 Theorem: In any μ-perfect intuitionistic fuzzy graph or υ-perfect intuitionistic fuzzy graph, the strength of connectivity $(CONN_{G\mu}(x, y), CONN_{G\upsilon}(x, y)) = (\mu_2(x, y), \upsilon_2(x, y))$ for all x, y ∈ V.

Proof:

Let G be a μ-perfect intuitionistic fuzzy graph, the strength of connectivity $(CONN_{G\mu}(x, y), CONN_{G\upsilon}(x, y)) = (1, 0)$ for all $x, y \in V$. Also the membership value and non membership value of each edge is (1, 0). Hence $(CONN_{G\mu}(x, y), CONN_{G\upsilon}(x, y)) = (\mu_2(x, y), \upsilon_2(x, y))$ for all $x, y \in V$.

Let G be a υ-perfect intuitionistic fuzzy graph, the strength of connectivity $(CONN_{G\mu}(x, y), CONN_{G\upsilon}(x, y)) = (0, 1)$ for all $x, y \in V$. Also the membership value and non membership value of each edge is (0, 1). Hence $(CONN_{G\mu}(x, y), CONN_{G\upsilon}(x, y)) = (\mu_2(x, y), \upsilon_2(x, y))$ for all $x, y \in V$.

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