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ON THE EXISTENCE OF ELECTRIC CHARGE

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ABSTRACT

It has been established that electromagnetic waves propagate with the speed of light. The reason is that the electrostatic fields as well as magnetic fields propagate with this speed. Both types of objects, waves as well as static field, contain and transport energy. Therefore it is possible to calculate how much energy and how much energy density a source of field emits into the space. Calculations show that this energy is not zero for elementary particles as well as macroscopic spheres. The calculation is presented in this paper. This leads to a principle problem that has not been answered to date: From where does the static charge obtain the energy, which it permanently emits? The paradox has a second string in its bow: If we allow the trace of a specified element of volume containing an electric field on its way through the space, we will notice that its contents of field energy decrease during time. But where does this effluent energy go?

Keywords: *Electromagnetic waves, Hertzian emitter, Biot Savart's law the paradox,*

INTRODUCTION

Structure of the chapter

The article is arranged into two parts. In the first part, the central ideas are presented as an overview but without detailed explanation. In the second part, some of the ideas are explained in detail.

Part 1: Overview of the contents

1. Statement

Electrostatic and magneto static fields propagate with the speed of light. They have the same speed of propagation as electromagnetic waves. Static fields cannot carry forward their interaction instantaneously – they are also restricted to the speed of light. Of course this upper limit for the speed is clear by principle. But we also know that the speed of propagation of the static fields really reaches the speed of light (and not a lower speed). This can be understood with the help of the example of the Hertzian dipole emitter, to which further explanation exists in detail number 1.

2. Statement

The procedure how an electric charge emits a static field can be compared with the emission of a gravitational field from a star in the universe. Both fill the space successively beginning with the moment of their birth – and both do it with the speed of light.

On the basis of the first statement that electrostatic fields propagate with the speed of light we perform a thought experiment. We consider a static charge which has been switched on at the time $t = t_o$, and from that moment on, it emits an electrostatic field. This could be a sphere with a conducting surface, on to which the charge has been brought at the moment $t = t_o$. But the field source can also be an elementary particle, for which the field has been switched 'on' at the moment of its genesis, which can be, for instance, the moment of its producing in a laboratory experiment, or, for instance, it may be the beginning of the universe.

3. Statement question

From where does the energy emitted with the field originate? Let us now turn our attention to a charge with spherical symmetry sometime after the field had been switched 'on'. It permanently emits an electrostatic field which contains energy. The problem is, from where does it get this energy? It does not contain a source of energy and it obviously does not convert mass into energy. Elementary particles (for example, electrons) do not alter their mass during time continuously. This is the first paradox of the existence of element charge (as sources of electrostatic fields). A way out of this paradox might be discussed with the assumption that every electric charge is being permanently supplied with energy from somewhere, but there is no serious indication for such an assumption in physics.

The amount of energy which an electrically charged particle emits per time interval is calculated in detail no. 2. But this calculation leads to a second paradox of the existence of electric charge. If we look again at an electric charge with radial symmetry (for instance, a charged sphere or a point charge), which emits the electrostatic field with spherical isotropy, we can select a concentric spherical shell of finite thickness and calculate the energy of the electrostatic field within this shell. During time this shell increases its radius, and we can trace it when propagating. Because of the conservation of energy, we expect that the field energy within the object remains constant during time. But the calculations show that this is not the case. The calculations show that the empty space reduces the energy within the given spherical shell. This means that the space expels the energy out of the spherical shell. But where does this energy go? The calculation to which this statement refers, is part of detail no. 2.

Now we can combine both paradoxes. On the one hand, the charge itself as the source of the field should be supplied permanently with energy; on the other hand the emanating field should give permanent energy to the space, but energies should not compensate each other completely, because the electrostatic field still contains some energy. By the way the definition of the classical electrons radius goes back to the energy of its electrostatic field.

4. Statement

Analogy with gravitation. It should be noticed that both paradoxes can be found in the static field of gravitation, at least in Newtón's conception. This field also has a static field energy with its energy density, and they also propagate into the space with the speed of light. From this point of view, the paradox of existence of electrostatic charge is completely analogous to the paradox of the existence of gravitation mass. Further explanations are to be found in detail no. 3.

Part 2: Explanation of details

1. The speed of propagation of electrostatic fields

It is obvious that sources of electromagnetic fields and electromagnetic waves permanently emit energy, because the fields as well as the waves contain energy. This argument would already be sufficient to verify the existence of the first paradox, but we want to look at it a bit more precisely, because a more detailed view will bring us later to the second paradox. The consideration begins with a thought experiment in which we can switch 'on' and 'off' electrostatic fields and magnetic fields. The mechanism for this switching operation can easily be imagined for the example of a magnetic field, because it is easy to switch 'on' and 'off' an electric current. A possible mechanism to switch 'on' and 'off' an electrostatic field could be the process of separating differently charged particles from each other inside a closed metal shielding box and taking part of the charge to the outside of the box. But a mechanism, for which a real example exists, in an oscillation pair of electric charges, which acts as a Hertzian dipole emitter. Its mechanism can be understood as follows: In the moment in which two electric charges of the same absolute value but of opposite algebraic signs are located at the same position, their electrostatic fields compensate each other exactly. In this moment we regard the electric charge to be switched off, because from every place in the space no charge is observable. As soon as both charges get different positions, as they do when they oscillate with regard to each other, each of them generates an electric field, and as long as both have different positions, they produce different field strengths at every point in the space. The superposition of both these fields is different from zero. During this time of the oscillation, the charge can be regarded to be switched 'on'. Only during those moments when their positions coincide, the electric field is switched 'off'. In a similar way, both charges generate a magnetic field as long as they are moving, but in the reversal point of the oscillation, when the movement stops for a moment, no magnetic field is emitted.

And all these fields, which are already generated and emitted, propagate into the space with the speed of light, without paying any attention to the question, whether there is some other field emitted earlier or later or not. As long as the oscillation of both the charges is going on, electric and magnetic fields with alternating field strength are emitted and propagate into the space. This is the typical explanation of the Hertzian dipole emitter with its characteristics.

Normally on the basis of these considerations, the emission characteristics of the Hertzian dipole emitter follow from the knowledge of the positions and the speed of the electrical charges as a function of time during their oscillation by calculating the electrostatic and the magnetic field strength at every time and every point in the space (according to Coulomb's law and according to Biot-Savart's law) and taking the speed of propagation of these fields into account, which is the speed of light.

In addition to the calculation, there is also a plausible illustration for the emission characteristics of the Hertzian dipole emitter.

In that part of the oscillation in which the field sources are on a movement from the common centre away, the field sources follow the emitted field strength.

In the moment we regard the electric charge to be switched off, because from every place in the space no charge is observable. As soon as both charges get different positions, as they do when they oscillate with regard to each other, each of them generates an electric field, and as long as both have different positions, they produce different field strengths at every point in the space. The superposition of both these fields is different from zero.

In the part of the oscillation in which the field sources are on a movement from the common centre away, the field sources follow the emitted field (which is also going from the common centre away) and by that they enhance the electric field strength. In the opposite part of the period of the oscillation, in which the field sources are on their movement towards the common centre, they go away from the emitted field (resp. they move towards the field into the opposite direction) and by that they supply less field strength per volume to the outside of the oscillation (and the field into the opposite direction is less strong because of the distance from the common centre).

The magnetic field is maximal in those moments in which the field sources pass the common centre, because in those moments the speed of the field sources reaches its maximum. Contrary to that, the moments of maximal deflection correspond to the minimal speed of movement of the field sources (in these moments their speed is zero), so that at this time no magnetic field is generated. By the way the phasing between the electric and the magnetic field occurring at electromagnetic waves is also explained from this consideration.

2. Calculation of the emitted energy and the propagation of this energy into space

Electric charges with spherical symmetry emit the field with spherical symmetry, and so it should be imaginable to calculate the field energy which the surface emits per time interval. For charges spheres with finite size, this way of calculation would be fine, but for punctiform charges it is not possible to define their surface or radius. One of them is the electron. Its classical radius ($r_{klass.}$ =2.8179...$10^{-15}m$) is defined by the means of its field energy but the value is in contradiction to the observations of scattering experiments, which lead to an upper limit of the electron's radius below $10^{-18}m$. Because it is not helpful to deal with this problem here, we surround the field source 'Q' with a spherical shell and calculate the energy passing the shell. The inner radius of the shell is not important, because energy coming from the field source has to pass the shell. This is the case for elementary particles in the same way as for macroscopic spheres. For the illustration of this statement please look to Fig. 1. The interpretation is the following.

We start our consideration at the time $t=0$ at which the field has just filled the inner shell with the radius x_1. Let us compare this with a moment $\Delta t > 0$ following a bit later. At this moment the field reaches the shell with the radius $x_1 + c.\Delta t$ (c= speed of light), so that during the time interval Δt the emitted energy has the same amount as the energy within the shell from x_1 to $x_1 + c.\Delta t$ because the total energy of the total field was enhanced just by this amount. This amount of energy, which is not zero, can only by generated by the source in the centre of the sphere, because there is no other source. This clarifies that the question about the origin of this energy is valid and leads to the first paradox. Subsequent to Fig.1 the emitted energy will be calculated.

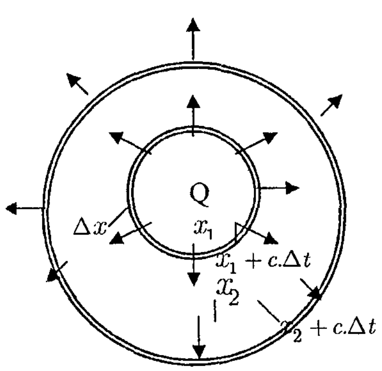

Fig. 1: *Illustration of a spherical shell which contains a certain amount of field energy. The sense of this construction is to trace the field energy when passing the empty space.*

The calculation of the amount of energy, which must be done to understand the paradoxes, is following now:

- The field strength produced by a charge Q with radial symmetry (i.e. a punctiform charge or a charge with spherical symmetry) is according to Coulomb's law $\vec{E}(\vec{r}) = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{r^3} \cdot \vec{r}$, where the centre of the charge is located in the origin of coordinates and $\vec{r}$ is the position of an arbitrary point in the space at which the field strength shall be determined.

- If we write $\vec{r}$ in spherical coordinates with $\vec{r} = (r, \vartheta, \varphi)$, the absolute values of the field strength are dependant not on the direction of $\vec{r}$ but only on the absolute value of $r = |\vec{r}|$, namely $E = |\vec{E}| = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{r^2}$.

- The energy density of the electric field is $u = \frac{\epsilon_0}{2} \cdot |\vec{E}|^2$.

- Consequently the energy density of the field produced by a charge with spherical symmetry is $u = \frac{\epsilon_0}{2} \cdot |\vec{E}|^2 = \frac{\epsilon_0}{2} \cdot \left(\frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{r^2} \right)^2 = \frac{Q}{32\pi^2 \epsilon_0 r^4}$.

- The energy within the spherical shell from x_1 to $x_1 + c.\Delta t$ can now be calculated as the volume integral.

$$E_{\substack{inner\\shell}} = \int_{\substack{spherical\\shell}} u(\vec{r})dV = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \int_{r=x_1}^{x_1+c.\Delta t} \frac{Q}{32\pi^2 \epsilon_0 r^4} . r^2 . \sin(\vartheta) dr d\vartheta d\varphi$$

$$= \frac{Q}{32\pi^2 \epsilon_0} . \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \underbrace{\int_{r=x_1}^{x_1+c.\Delta t} \frac{1}{r^2} . dr}_{= \frac{c.\Delta t}{(x_1+c.\Delta t).x_1}} . \sin(\vartheta) d\vartheta d\varphi$$

$$= \frac{Q}{32\pi^2 \epsilon_0} . \frac{c.\Delta t}{(x_1 + c.\Delta t).x_1} . \underbrace{\int_{\varphi=0}^{2\pi} \underbrace{\int_{\vartheta=0}^{\pi} \sin(\vartheta) d\vartheta}_{=2} d\varphi}_{=4\pi}$$

$$= \frac{Q}{32\pi^2 \epsilon_0} . \frac{c.\Delta t}{(x_1 + c.\Delta t).x_1} . 4\pi = \frac{Q}{8\pi \epsilon_0} . \frac{c.\Delta t}{(x_1 + c.\Delta t).x_1}$$

Obviously this energy is not zero. This means that the source indeed emits energy permanently. This is the reason for the first paradox.

Let now the time elapse until it reaches t_2. The inner border of the observed spherical shell has now passed from x_1 to x_2 and the outer border from $x_1 + c.\Delta t$ to $x_2 + c.\Delta t$. With the distance $.\Delta t$ introduced in Fig.1, we find the inner and the outer border of the shell being at the radii $x_1 = x_1 + \Delta x$ respectively $x_2 + c.\Delta t = x_1 + \Delta x + c.\Delta t$. The spherical shell has enhanced its volume, but the field strength within this moving shell has been reduced (in accordance with Coulomb's law). If the empty space would allow the field energy just to pass by, the amount of energy within the outer shell $E_{outershell}$ should be the same as the amount of energy within the inner shell $E_{innershell}$. We want to check this:

$$E_{\substack{outer\\shell}} = \int_{\substack{sherical\\shell}} u(\vec{r})dV = \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi}\int_{r=x_2}^{x_2+c.\Delta t} \frac{Q}{32\pi^2 \in_0 r^4}.r^2.\sin(\vartheta)drd\vartheta d\varphi$$

$$= \frac{Q}{32\pi^2 \in_0}.\int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} \underbrace{\int_{r=x_1+\Delta x}^{x_1+\Delta x+c.\Delta t} \frac{1}{r^2}.dr}_{\frac{c.\Delta t}{(x_1+\Delta x+c.\Delta t).(x_1+\Delta x)}} .\sin(\vartheta)d\vartheta d\varphi$$

$$= \frac{Q}{32\pi^2 \in_0}.\frac{c.\Delta t}{(x_1+\Delta x+c.\Delta t).(x_1+\Delta x)}.\underbrace{\int_{\varphi=0}^{2\pi}\underbrace{\int_{\vartheta=0}^{\pi}\sin(\vartheta)d\vartheta}_{=2} d\varphi}_{=4\pi}$$

$$= \frac{Q}{32\pi^2 \in_0}.\frac{c.\Delta t}{(x_1+\Delta x+c.\Delta t).(x_1+\Delta x)}.4\pi = \frac{Q}{8\pi \in_0}.\frac{c.\Delta t}{(x_1+\Delta x+c.\Delta t).(x_1+\Delta x)}$$

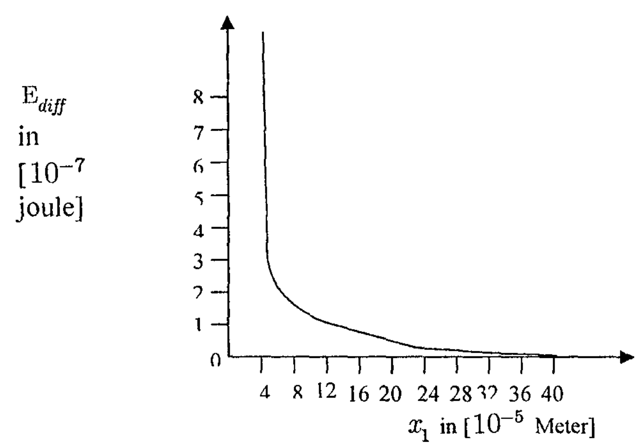

Fig. 2:
Plot of the energy difference which the spherical shell of our thought experiment lost when passing the empty space.
x_1 is inner radius of the shell. For the example of our calculation the input was taken as $\Delta t = 10^{-10}m$ and $\Delta t = 10^{-7}$., and the source of the field was an

Obviously the energy $E_{\substack{inner\\shell}}$ is more than the energy $E_{\substack{outer\\shell}}$. This means that the empty space decreases the energy of the shell. This proves the validity of the second paradox, which poses the question: Where does the difference of energy go? In order to complete the thoughts, Fig. 2 shows a plot of the difference of energy $E_{diff} = E_{\substack{inner\\shell}} - E_{\substack{outer\\shell}}$, which the spherical shell lost on its way Δx across the empty space, beginning from the radius x_1,

3. The analogy of the problem for the case of gravitation

At least in Newton's description of gravitation we see the following concept: The analogy of the paradox of the existence of electric charge with the paradox of the existence of gravitational mass can be understood by the means of the analogy of the propagation of static fields and waves.

The gravitational counterpart of the electrostatic fields is the fields of gravitation. The gravitational counterpart of electromagnetic waves are gravitational waves. Their existence is accepted today, and the work is in progress to detect them experimentally.

Also the mechanism of their generation can be understood in analogy with the generation of electromagnetic waves. In the electromagnetic case, oscillating charges can be responsible for periodic switching 'on' and 'off' the field, and by that they generate the waves. In the case of gravitation we know for instance the example of a double star (or other examples) rotating around a common centre of mass, and creating gravitational waves. The characteristic of the emitted field looks different from the characteristic created by a harmonic oscillation, but this does not influence the principle aspects of our logical track here. The important aspect is that it is generally accepted that gravitational waves together with gravitational fields propagate with the speed of light.

Thus the analogy with the mechanism and the formula of detail 2 is obvious. We just have to replace the energy density of the gravitational field, which is $u = \frac{1}{8\pi\gamma} \cdot |\vec{G}|^{2,S}$ (with γ = Newton's constant of gravitations and $\vec{G} = \gamma \cdot \frac{M}{r^3} \cdot \vec{r}$, where M is the gravitational mass) and put this energy density into volume integrals shown there. For the calculation, only the values of the constants in front of the integrals have to be demonstrated once more now. The crucial point is clear.

Also for gravitational fields the two paradoxes remain as open questions:

From where does the energy descend which every gravitational mass emits permanently?

Why does the empty space absorb energy from the gravitational field during propagation – and where does this energy go?

REFERENCE

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