

STELLA MARIS COLLEGE(AUTONOMOUS), CHENNAI-86.

(For candidates admitted after the year 2011-12)

M.PHIL DEGREE EXAMINATION - APRIL 2013

SUBJECT CODE: 11MT/RO/GT2 05

BRANCH I –MATHEMATICS

COURSE: OPTIONAL PAPER

PAPER : GRAPH THEORY

TIME: 3 HOURS

MAX MARKS:100

ANSWER ANY FIVE:

(5 x 20 = 100)

1. a) Prove that a graph is bipartite if and only if it has no odd cycle.  
b) Prove or disprove that every Eulerian bipartite graph has an even number of edges.  
c) Explain in detail Dijkstra's Algorithm. (7 + 5 + 8)
  2. a) Prove that for an  $n$ -vertex graph  $G$  (with  $n \geq 1$ ) the following are equivalent:  
i)  $G$  is connected and has no cycles.  
ii)  $G$  is connected and has  $n - 1$  edges.  
iii)  $G$  has  $n - 1$  edges and has no cycles.  
iv)  $G$  has no loops and has for each  $u, v \in V(G)$ , exactly one  $u$ - $v$  path.  
b) Find the number of spanning trees of a complete graph on 6 vertices. (12 + 8)
  3. a) Define perfect matching and Maximum matching and find the same for  $K_{3,3}$ .  
b) Determine the minimal size of a maximal matching in a cycle.  
c) State and prove Hall's theorem. (6 + 4 + 10)
  4. a) Define chromatic number and find the same for a cycle on 8 vertices and  $P_{10}$ .  
b) State and prove Brook's theorem.  
c) State any two applications of colouring problem. (6 + 10 + 4)
  5. a) Define crossing number and obtain the same for  $P(6,2)$ .  
b) State and prove Euler's formula.  
c) Prove that if  $G$  is a graph with fewest edges among all nonplanar graphs without Kuratowski subgraphs, then  $G$  is 3-connected. (5 + 7 + 8)
  6. a) State and prove the necessary and sufficient condition for a graph to be Eulerian.  
b) State and prove Dirac's condition for a Hamiltonian graph. (10 + 10)
  7. a) Define an Interconnection Network.  
b) Explain the basic principles of Network Design. (4 + 16)
  8. a) Draw a graph which can be modeled as Interconnection Network and explain its properties.  
b) Draw a de Bruijn graph and explain its properties. (10+10)
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