STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI-86

(For the academic year 2011-2012and there after)

Subject code : 11MT/RO/FA205

M.Phil DEGREE EXAMINATION APRIL 2013

ANSWER ANY **FIVE** QUESTIONS **(5x20)**

PAPER: **FUNCTIONAL ANALYSIS** MARKS : 100

TIME: 3 HRS

1a)Define a complete metric space; prove that L_p is complete.
b) State and prove the Banach contraction principle.

2a) State and prove the necessary and sufficient condition for a metric space to be Compact .
b) State and prove the theorem on existence and uniqueness of the solution of an integral equation.

3.a) State and prove the Inverse theorem on Banach space for a bounded linear operator.
b) Prove that the spectrum of a bounded linear operator is a closed set in the complex plane.

4.a) Prove that if T is a compact linear operator then for a nonzero λ, the range of $T_\lambda = T - \lambda I$ is closed .
b) State and prove the properties of Null space.

5. a) Define monotone sequence and state and prove its property.
b) State and prove the properties of product and sum of projections.

6. If n^{th} partial derivative of the function f exists in a nbd of the point $T_0 = (t_1^{(0)}, t_2^{(0)} \ldots t_n^{(0)})$ and if this derivative is continuous at T_0,then prove that the n^{th} partial difference derivative also exists at T_0 and that both the derivative coincide. Further show that the n^{th} partial derivative doesnot depend on the order of differentiation.

7. Define Frechet differential and weak differential. If the Frechet differential exists, then prove that the weak differential also exists, and they are equal.

8.a) Define Haar condition.
b) State and prove the Haar uniqueness theorem for best approximation.