

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086

M. Phil. DEGREE EXAMINATION APRIL 2013

DEPARTMENT OF MATHEMATICS

ADVANCED ALGEBRA

CLASS: M.PHIL.

MAX: 100 MARKS

CODE:11MT/RO/AA2 05

TIME: 3 HOURS

Answer any Five questions (Each question carries 20 marks)

1. State and prove Jacobson's density theorem on semi simple  $R$ -modules.
  2. Prove the structure theorem for semi simple Rings.
  3. i) Prove that an exact functor from  $\mathcal{T}$  to  $\mathcal{D}$  transforms every exact sequence in  $\mathcal{T}$  to an exact sequence in  $\mathcal{D}$ .  
ii) Define Projective module and free module. Prove that every free module is projective.
  4. i) State and prove Schanuel's lemma.  
ii) State and prove the equivalent conditions for a module to be projective.
  5. a) Define Injective module and give an example.  
b) State and prove Baer's Theorem.  
c) Prove that every  $Z$ -module can be embedded in an injective  $Z$ -module.
  6. a) Prove that every finitely generated projective  $A$ -module  $P$  is finitely presented.  
b) Prove that every projective module of rank  $n$  over a semi-local ring is free of rank  $n$ .
  7. State and prove Wedderburn's Structure theorem on semi-simple  $R$  – algebras
  8. i) If  $A$  is an  $R$ -algebra and  $M$  is a free right  $A$ -module, prove that
$$E_A(\oplus_n M) \cong M_n(E_A(M))$$
  
ii) State and prove Masche's Theorem.  
iii) Define a graded ring and give an example.
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