STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PE/FT24

M. Sc. DEGREE EXAMINATION, APRIL 2013
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : ELECTIVE
PAPER : FUZZY SET THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION –A

Answer all the questions: **5×2=10**

1. Define α - cut and strong α - cut of a fuzzy set and state any one of the properties of the α - cut of a fuzzy set A.
2. Define a normal fuzzy relation and give an example.
3. Define Yager class of fuzzy complements.
4. Define a fuzzy number.
5. Illustrate any application of fuzzy logic in day today life.

SECTION –B

Answer any five questions: **5×6=30**

6. Discuss the fuzzy equation $+X = B$.
7. Let $f: X \to Y$ be an arbitrary crisp function. Then, for any $A \in \mathcal{F}(X)$, prove that f fuzzified by the extension principle satisfies the equation $f(A) = \cup_{\alpha\in[0,1]} f({}_{\alpha+}A)$
8. Prove that every fuzzy complement has at most one equilibrium.
9. Find $A + B$ and $A - B$ if A and B are fuzzy numbers given by

$$A(x) = \begin{cases} 0 & for\ x \le -1 \, and\ x > 3 \\ (x+1)/2 & for -1 < x < 1 \\ (3-x)/2 & for\ 1 < x \le 3, \end{cases}$$

$$B(x) = \begin{cases} 0 & for\ x \le 1 \, and\ \ x > 5 \\ (x-1)/2 & for\ 1 < x \le 3 \\ (5-x)/2 & for\ 3 < x \le 5. \end{cases}$$

10. Explain the different methods for designing fuzzy control rules.
11. Represent the concept of "middle aged person" by a fuzzy set using trapezoidal membership function, considering the persons with age between 35 and 45 as middle aged persons.
12. Prove that a fuzzy set A is convex if and only if $A(\lambda x_1 + (1-\lambda)x_2) \ge \min[A(x_1), A(x_2)]$.

SECTION –C

Answer any three questions: **3×20=60**

13. (a) Explain the standard operations of union, intersections, complements of fuzzy sets with examples.

(b) Let $A, B \in \mathcal{F}(X)$. Then prove that the following the statements hold for all $\alpha, \beta \in [0,1]$.

(i) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ and ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B^{\alpha}$

(ii) ${}^{\alpha}(\bar{A}) = {}^{(1-\alpha)+}\bar{A}$

14. (a) Let $f : X \to Y$ be an arbitrary crisp function. Then, for any $A \in \mathcal{F}(X)$, and all $\alpha \in [0,1]$, prove that the following properties of f fuzzified by the extension principle hold.

$(i).\ {}^{\alpha+}[f(A)] = f({}^{\alpha+}A).$

$(ii).\ {}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A).$

Also prove that equality in (ii) does not hold in general.

(b) Explain fuzzy relational equations.

15. Given a t-norm i and an involutive fuzzy complement c, prove that the binary operation u on [0,1] defined by $u(a,b) = c(i(c(a), c(b))$, for all $a, b \in [0,1]$ is a t-conorm such that (i, u, c) is a dual triple.

16. Let $*$ denote any of the four basic arithmetic operations namely $+, -, .\ , /$ and let A and B denote continuous fuzzy numbers. Then prove that the fuzzy set $A * B$ defined by $(A * B)(x) = \sup_{z=x*y} \min[A(x), B(y)]$ is a continuous fuzzy number.

17. Discuss the applications of fuzzy theory in industries.