STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12 & thereafter)

SUBJECT CODE : 11MT/PC/TO24

M. Sc. DEGREE EXAMINATION, APRIL 2013
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5×2=10**

1. Define nowhere dense set.
2. Define limit point of a set.
3. Define Bolzano-Weierstrass property.
4. Define a Hausdorf space.
5. Is every connected space locally connected? Justify your answer.

SECTION – B

Answer any five questions: **5×6=30**

6. If X and Y are metric spaces and f is a mapping of X into Y, then show that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.
7. If X is a topological space and $A \subseteq X$, then prove that $x \in \bar{A}$ if and only if every neighbourhood of x intersects A.
8. Show that a closed subspace of a compact space is compact.
9. State and prove Lebesgue's covering lemma.
10. Show that every compact Hausdorff space is normal.
11. Prove that the continuous image of connected space is connected.
12. If A is a connected subspace of a topological space X and B is a subspace of X such that $A \subseteq B \subseteq \bar{A}$, then show that B is connected.

SECTION – C

Answer any three questions: **3×20=60**

13. a) State and prove the Baire's Theorem.
    b) If X and Y are metric spaces and f is a mapping of X onto Y, then prove that f is continuous at x_0 if and only if $x_n \longrightarrow x_0 \Rightarrow f(x_n) \longrightarrow f(x_0)$.

14. a) Show that every second countable space is separable but not conversely.
    b) If $f: X \longrightarrow Y$ be a function from a topological space into a topological space, then prove that f is continuous if and only if $f^{-1}(K)$ is closed for each closed K in Y.

15. a) State and prove Heine Boral theorem.
    b) State and prove Tychonoff theorem.

16. State and prove Urysohn's lemma.

17. a) Show that a subspace of the real line is connected if and only if it is an interval.
    b) Prove that $\mathbb{R}^n$ and $\mathbb{C}^n$ are connected.