STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011–12)

SUBJECT CODE: 11MT/PC/PD44

M. Sc. DEGREE EXAMINATION, APRIL 2013
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS **(5 x 2 = 10)**

1. Formulate the partial differential equation by eliminating the arbitrary constants a and b from z = x + ax^2y^2 + b
2. Mention the three classifications of the 2nd order partial differential equation $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ based on the conditions on the functions R, S and T.
3. State the boundary value problem of the first kind.
4. Write down the three dimensional diffusion equation in cylindrical coordinates.
5. State the Green's identity.

SECTION – B

ANSWER ANY FIVE QUESTIONS ONLY **(5 x 6 = 30)**

6. Eliminate the arbitrary function f from the relation z = xy + $f(x^2 + y^2)$.
7. Using Charpit's method, solve the partial differential equation $(p^2 + q^2)y = qz$.
8. Show that u = $f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$.
9. Solve the equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$.
10. Show that the two dimensional Laplace equation $\nabla_1^2 V = 0$, in the plane polar coordinates r and θ has the solution of the form $(Ar^n + Br^{-n})e^{\pm in\theta}$ where A and B are constants.
11. State transmission line problem with special reference to current loss in a cable of length l carrying current with resistance R, inductance L, capacitance C and leakance G. Derive the Telephone and Telegraph equations.
12. By separating the variables, show that the one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ has solution of the form $Ae^{\pm inx \pm inct}$.

SECTION – C

ANSWER ANY THREE QUESTIONS ONLY **(3 x 20 = 60)**

13. a. Use Jacobi's method to solve $(p^2x + q^2y) = z$.

    b. Find the surface which is orthogonal to the one parameter system z $= cxy(x^2 + y^2)$ and which passes through the hyperbola $(x^2 - y^2) = a^2,\ z = 0$.

14. a. Solve $\nabla_1^2 z = e^{-x}\cos y$, which tends to zero as $x \to \infty$ and has the value $\cos y$ when $x = 0$.

    b. Reduce the equation $(n-1)^2u_{xx} - y^{2n}u_{yy} = ny^{2n-1}u_y$ to a canonical form and find its general solution, where n > 1.

15. a. Express the Laplace equation in cylindrical coordinates and derive the general solution of the same.

    b. State and prove the Maximum-Minimum principle.

16. a. State and derive D'Alembert solution of one dimensional wave equation.

    b. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0\, sin^3(\frac{\pi x}{l})$. It is released from rest from this position. Find the displacement $y(x,t)$.

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