STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12)

SUBJECT CODE : 11MT/PC/DG44

M. Sc. DEGREE EXAMINATION, APRIL 2013
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : DIFFERENTIAL GEOMETRY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL QUESTIONS : **(5 X 2 = 10)**

1. Define parameterised curve in $\mathbb{R}^n$.
2. Define a smooth surface.
3. When is a map said to be conformal?
4. Define geodesic curvature.
5. Define Gaussian curvature.

SECTION – B

ANSWER ANY FIVE QUESTIONS : **(5 X 6 = 30)**

6. If γ is a unit-speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion then prove that γ is a circle.
7. Given U and $\bar{U}$ are open subsets of $\mathbb{R}^2$ and $\sigma: U \to \mathbb{R}^3$ is a regular surface patch and $\varphi: \bar{U} \to U$ a bijective smooth map with smooth inverse map $\varphi^{-1}: \bar{U} \to U$ then show that $\bar{\sigma} = \sigma \circ \varphi: \bar{U} \to \bar{U}$ is a regular surface patch.
8. If $\sigma: U \to \mathbb{R}^3$ be a patch of a surface S containing a point P of S and (u, v) are coordinates in U, then prove that the tangent space to S at P is the vector subspace of $\mathbb{R}^3$ spanned by the vectors σ_u and σ_v.
9. Prove that $\|\sigma_u \times \sigma_v\| = (EG - F^2)^{1/2}$.
10. Show that any tangent developable is isometric to a plane.
11. Obtain the second fundamental form for the surface of revolution
 $\sigma(u, v) = (f(u)\cos v,\ f(u)\sin u, g(u))$.
12. If $\sigma(u, v)$ be a surface patch with first and second fundamental forms then prove that $K = \frac{LN - M^2}{EG - F^2}$.

SECTION – C

ANSWER ANY THREE QUESTIONS : **(3 X 20 = 60)**

13. a) Show that a parameterised curve has a unit-speed reparametrisation if and only if it is regular.

b) Prove that the curvature of a regular curve $\gamma(t)$ is $\mathbb{R}^3$ is

$$k = \frac{\|\ddot{\gamma} \times \dot{\gamma}\|}{\|\dot{\gamma}\|^3} \quad \text{where} \quad . = \frac{d}{dt}.$$

14. a) Explain the concept of a smooth map.

b) If $f: S_1 \to S_2$ be a diffeomorphism and σ_1 is an allowable surface patch on S_1, then show that $f \circ \sigma_1$ is an allowable surface patch on S_2.

15. Prove that a diffeomorphism $f: S_1 \to S_2$ is an isometry if and only if, for any surface patch σ_1 of S_1, the patches σ_1 and $f \circ \sigma_1$ of S_1 and S_2 respectively, have the same first fundamental form.

16. a) State and prove Meusnier's theorem.

b) State and prove Euler's theorem.

17. Show that any point of a surface of constant Gaussian curvature is contained in a patch that is isometric to part of a plane, a sphere or a pseudo sphere,