STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2009–10 & thereafter)

SUBJECT CODE: MT/PC/GT44

M. Sc. DEGREE EXAMINATION, APRIL 2011
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIIONS: **5X8=40**

1. a) Define an induced subgraph of a graph and illustrate with an example.
   b) In any graph, prove that the number of vertices of odd degree is even.
   (3+5)

2. a) State any four properties of a tree.
   b) Define a spanning tree of a graph. What is the spanning tree of a tree?
   (4+4)

3. a) Define the connectivity $K(G)$ of a graph G. When is $K(G) = 0$?
   b) Show that a graph G with $\gamma \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.
   (3+5)

4. a) Give an example of a graph which is neither Eulerian nor Hamiltonian.
   b) State and prove a necessary and sufficient condition for a graph to Eulerian.
   (2+6)

5. a) Prove that a matching M in G is a maximum matching if and only if G contains no $M -$ augmenting path.
   b) Define a covering of a graph and find the same for $K_{1,n}$.
   (5+3)

6. a) Prove that $\alpha + \beta = \gamma$ with usual notation.
   b) Define chromatic number of a graph and find the same for the Petersen graph.
   (5+3)

7. a) State and prove Euler's formula on planar graphs.
   b) Define an interconnection network. State any two properties of a Hypercube.
   (5+3)

SECTION – B

ANSWER ANY THREE QUESTIONS: (3 X20 = 60)

8. a) Show that a graph is bipartite if and only if it contains no odd cycles.
   b) Prove that a vertex v of a tree G is a cut vertex if and only if $d(v) > 1$.
   c) Every vertex of a tree is a cut vertex – True or False. Justify.

(8+8+4)

9. a) With usual notations prove that $K \leq K' \leq \delta$.
   b) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq {}^{\gamma}/_{2}$, then prove that G is Hamiltonian.
   c) Define domination number of a graph and find the same for cycles C_n.

(8+8+4)

10. a) For a bipartite graph with bipartition (X, Y), state and prove a necessary and sufficient condition for a matching to saturate every vertex in X.
    b) State and prove Brook's theorem.

(10+10)

11. a) If G is k −critical, then prove that $\delta \geq k - 1$.
    b) State and prove five colour theorem.

(8+12)

12. a) Explain any four basic principles of network design.
    b) Give two equivalent definitions of a hypercube network.
    c) Define a circulant network and state any four of its properties.

(8+6+6)