STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2009–10 & thereafter)

SUBJECT CODE: MT/PC/FA44

M. Sc. DEGREE EXAMINATION, APRIL 2012
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : FUNCTIONAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY FIVE QUESTIONS: **(5 X 8 = 40)**

1. If N is a normed linear space, show that the closed unit sphere S^* in N^* is compact in $weak^*$ topology.

2. Stare and prove uniform boundedness theorem.

3. If M is a proper closed linear subspace of a Hilbert space H, show that there exists a non zero vector z_0 in H such that $z_0 \perp M$.

4. State and prove Schwarz inequality and hence deduce that inner product in a Hilbert Space is jointly continuous.

5. If N_1 and N_2 are normal operators on a Hilbert Space H with the property that either commutes with the adjoint of the other, show that $N_1 + N_2$ and $N_1\, N_2$ are normal.

6. If T is an operator on a Hilbert Space H and $[T]$ is the matrix corresponding on an ordered basis B. If T_1, T_2 are operators on H, show that $[T_1 + T_2] = [T_1] + [T_2]$ and $[T_1T_2] = [T_1][T_2]$.

7. Let A be a Banach Algebra and $x \in A$. Show that the spectral radius $r(x) = \lim \|x^n\|^{\frac{1}{n}}$.

SECTION – B

ANSWER ANY THREE QUESTIONS: **(3 X 20 = 60)**

8. State and prove open mapping theorem.

9. If $\{e_i\}$ is an orthonormal set in a Hilbert Space H and if $x \in H$, show that $x - \sum (x, e_i) e_i \perp e_j$ for each j.

10. a) If T is an operator on a Hilbert Space H, for which $(Tx, x) = 0$ for all x, show that $T = 0$.

b) If T is an operator on a Hilbet Space H, show that T is normal if and only if its real and imaginary parts commute.

c) If T is an operator on H show that the following conditions are equivalent.

(i) $T * T = I$

(ii) $(Tx, Ty) = (x, y)$ for all $x, y \in H$

(iii) $\|Tx\| = \|x\|$ for all x. (7+6+7)

11. a) If T is an arbitrary operator on a Herbert Space H, show that the eigen values of T constitute a non empty finite subset of the complex plane.

b) Show that two matrices in A_n are similar if and only if they are matrices of a single operator on H relative to different bases. (8+12)

12. a) Show that the set of all regular elements in a Banach algebra is an open set.

b) Show that every boundary point in the set of all singular elements is a topological divisor of zero.

c) Show that spectrum $\sigma(x)$ is non empty for each $x \in A$.