STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12)

SUBJECT CODE : 11MT/PE/TS24

M. Sc. DEGREE EXAMINATION, APRIL 2012
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : ELECTIVE
PAPER : TENSOR ANALYSIS AND SPECIAL THEORY OF RELATIVITY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION –A

Answer all the questions: **5×2=10**

1. Show that $\frac{\partial x^p}{\partial \bar{x}^q} \frac{\partial \bar{x}^q}{\partial x^r} = \delta_r^p$.
2. Prove that $g_{21}G(3,1) + g_{22}G(3,2) + g_{23}G(3,3) = 0$.
3. If the covariant force acting on a particle is given by $F_x = \frac{-\partial v}{\partial x^k}$ where $V(x^1, \ldots, x^v)$ is the potential energy, show that $\frac{d}{dt}\left(\frac{\partial L}{\partial i^k}\right) - \frac{\partial L}{\partial x^k} = 0$ where $L = T - V$.
4. Write down Galilean transformation equations.
5. Define invariant interval.

SECTION –B

Answer any five questions: **5×6=30**

6. If A_r^{pq} and B_r^{pq} are tensors prove that their sum and difference are also tensors.
7. Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other is skew symmetric in a pair of covariant and contravariant indices.
8. Show that $\frac{\partial}{\partial x^m}\left[g^{ik} g_{ij}\right] = 0$ and then $\frac{\partial g^{pq}}{\partial x^m} = -g^{pn}\begin{Bmatrix} q \\ mn \end{Bmatrix} - g^{qn}\begin{Bmatrix} p \\ mn \end{Bmatrix}$.
9. Prove that a necessary condition that $I = \int_{t_1}^{t_2} F(t_1 x_1\, \dot{x})dt$ be an extremum is that $\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right) = 0$.
10. Explain principle of relativity.
11. Explain proper time.
12. Obtain addition of velocities.

SECTION –C

Answer any three questions: **3×20=60**

13. a) A covariant tensor has components $xy,\ zy - z^2,\ xz$ in rectangular coordinates. Find its covariant components in spherical coordinates.

    b) Define $g^{jx} = \frac{G(j,k)}{g}$ where $G(j,k)$ is the cofactor of g_{jk} in the determinant $g = |g_{jk}| \neq 0$ prove that $g_{jk}g^{pk} = \delta_j^p$.

14. Derive transformation laws for the Christoffel symbols of

    (a) the first kind (b) the second kind

15. a) Obtain the geodesics in a Riemannian Space.

    b) If A^p and A^p are tensor show that $A_{p,q} = \frac{\partial A_p}{\partial x^q} - \begin{Bmatrix} S \\ pq \end{Bmatrix} A_s$ is tensor.

16. Define work and energy in tensor form and derive the Lagrange's equation for a force system to be conservative.

17. Obtain Lorentz transformation equations.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄