STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12)

SUBJECT CODE : 11MT/PE/FT24

M. Sc. DEGREE EXAMINATION, APRIL 2012
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : ELECTIVE
PAPER : FUZZY SET THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION –A

Answer all the questions: **5×2=10**

1. Define Union and Intersection of fuzzy sets.
2. Define Max-min composition of two Binary fuzzy relations.
3. For each $a \in [0,1]$, show that ${}^{d}a = c(a)$ if and only if $c\big(c(a)\big) = a$.
4. Define addition, subtraction and multiplication of fuzzy intervals.
5. Mention the use of fuzzy sets in Engineering.

SECTION –B

Answer any five questions: **5×6=30**

6. Show that a fuzzy set A on R is convex if and only if for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$, $A[\lambda x_1 + (1-\lambda)x_2] \geq \min\,[A(x_1), A(x_2)]$.

7. For any two fuzzy sets A and B, show that ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ and ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$ for any $\alpha \in [0,1]$.

8. Let $f: X \to Y$ be an arbitrary crisp function. If A is any fuzzy set of X, then show that ${}^{\alpha+}[f(A)] = f[{}^{\alpha+}A]$ and ${}^{\alpha}[f(A)] \supseteq f[{}^{\alpha}A]$ for any $\alpha \in [0,1]$.

9. Discuss the fuzzy relational equation with an example.

10. Define fuzzy complement. If c is a continuous fuzzy complement, then show that c has a unique equilibrium.

11. For the fuzzy numbers A and B given below, determine the fuzzy numbers $A + B,\ A - B$ and $A.B$ where

$$A(x) = \begin{cases} 0 & \text{for } x \leq -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 < x \leq 1 \\ (3-x)/2 & \text{for } 1 \leq x \leq 3 \end{cases}$$

$$B(x) = \begin{cases} 0 & \text{for } x \leq 1 \text{ and } x > 5 \\ (x-1)/2 & \text{for } 1 < x \leq 3 \\ (5-x)/2 & \text{for } 3 \leq x \leq 5 \end{cases}$$

12. Describe the mathematics of fuzzy controller.

SECTION –C

Answer any three questions: **3×20=60**

13. a) Write the features that are responsible for the Paradigm shift from the classical set theory.
    b) Explain the basic concepts of different fuzzy sets.

14. a) Determine the max-min composition and max-product composition for the following Binary Relation.

$$P(X,Y) = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix} \text{ and}$$

$$Q(Y,X) = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix}$$

    b) Explain Crisp and Fuzzy relations with suitable example.

15. a) Show that $\lim_{\omega\to\infty} \min[1, (a^{\omega} + b^{\omega})^{\frac{1}{\omega}}] = \max(a, b)$
    b) For all $a, b \in [0,1]$. Show that (i) $u(a,b) \geq \max(a,b)$
    (ii) $i(a,b) \leq \min(a,b)$.

16. a) Derive a necessary and sufficient condition for functions to be membership function of fuzzy numbers.
    b) If A and B are continuous fuzzy numbers then show that $A * B$ given by

$$(A * B)(z) = \underbrace{\sup}_{z=x*y} \min[A(x), B(y)]$$

    is a continuous fuzzy number where the operators $* \in \{+, -, ., /\}$.

17. Discuss the application of Fuzzy Mathematics in Industry.

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄