SUBJECT CODE : 11MT/PE/BM24

M. Sc. DEGREE EXAMINATION, APRIL 2012
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : ELECTIVE
PAPER : BASIC MATHEMATICAL METHODS
TIME : 3 HOURS   MAX. MARKS : 100

SECTION –A

Answer all the questions: 5×2=10

1. Show that the matrix $A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.

2. If $y = x^{x^{x\ldots\infty}}$, find $\frac{dy}{dx}$.

3. Evaluate $\int sin^2 x \, dx$.

4. The rate of monthly salary of a person increases annually in A.P. It is known that he was drawing Rs.200 a month during the 11th year of service and Rs.380 during the 29th year. Find his starting salary and the rate of annual increment.

5. Solve $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$.

SECTION –B

Answer any five questions: 5×6=30

6. If $= \begin{bmatrix} 2 & 2 & 5 \\ 5 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 7 \\ 2 & 5 \\ 3 & -2 \end{bmatrix}$ verify that $(AB)^T = B^T A^T$.

7. Prove that if $y = sin\,(m\,sin^{-1}x)$, then $(1 - x^2)y_2 - xy_1 + m^2 y = 0$.

8. Differentiate with respect to x, (a) $\frac{1+x^2}{1-x^2}$; (b) $xe^x sinx$.

9. Evaluate $\int e^{ax} sinbx \, dx$.

10. Given $sin45 = 0.7071, sin50 = 0.7660, sin55 = 0.8192, sin60 = 0.8660$. Find $sin52$ using suitable Interpolation formula.

11. Three numbers whose sum is 18 are in A.P. If 2, 4, 11 are added to them respectively. The resulting numbers are in G.P. Determine the numbers.

12. Solve $\frac{dy}{dx} + ycosx = \frac{sin2x}{2}$.

SECTION –C

Answer any three questions: 3×20=60

13. a) Solve by matrix method $2x + 4y + z = 5$; $x + y + z = 6$; $2x + 3y + z = 6$.

b) If $A = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$. Show that A satisfies $(A - 10I)(A - I) = 0$. Hence find A^3.

(10+10)

14. a) Find the maxima and minima of $x^3 - 18x^2 + 96x + 4 = 0$.

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{x+y+z}$.

c) Find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(4+8+8)

15. a) Find $y(10)$ using Lagrange's Interpolation formula from the given data:

x	5	6	9	11
y	12	13	14	16

b) Evaluate $\int \frac{dx}{x^2+4x+13}$.

c) Prove that $\int_0^{\pi/4} \log(1 + tanx)\, dx = \frac{\pi}{8} log2$.

(6+6+8)

16. a) Insert three geometric means between 3 and $\frac{3}{16}$.

b) Sum to infinity the series $1 + \frac{1+2}{1!} + \frac{1+2+2^2}{2!} + \frac{1+2+2^2+2^3}{3!} + \cdots \infty$

c) Expand $f(x) = \begin{cases} -x, & -\pi < x \leq 0 \\ x, & 0 < x < \pi \end{cases}$ only as the cosine series.

(6+6+8)

17. Solve a) $xdy - ydx = \sqrt{x^2 + y^2}\, dx$.

b) $(D^2 - 5D + 6)y = e^{4x}$.

c) $(D^2 - 2D - 8)y = 4cos2x$.

(6+6+8)

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