STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2011-12)

SUBJECT CODE : 11MT/PC/TO24

M. Sc. DEGREE EXAMINATION, APRIL 2012
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : TOPOLOGY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5×2=10**

1. Define a Cauchy sequence.
2. Define the closure of a set A, in a topological space X.
3. Define a compact space.
4. Define a Hausdorff space.
5. Show that the continuous image of a connected space in connected.

SECTION – B

Answer any five questions: **5×6=30**

6. State and prove Cantor's Intersection theorem.
7. If f is a mapping from one metric space X to another Y, then show that f is continuous if and only if the inverse image of an open set is open.
8. State and prove Lindolöf theorem.
9. Let X be a topological space and $A \subseteq X$. Then $x \in \bar{A}$ if and only if every neighbourhood of x intersects A.
10. Stare and prove Tychonoff's theorem
11. Prove that every compact Hausdorff space is normal.
12. Show that a subspace of the real line $\mathbb{R}$ is connected if and only if it is an interval.

SECTION – C

Answer any three questions: **3×20=60**

13. a) If Y is a subspace of a complete metric space X then prove that Y is complete if and only if it is closed.

    b) Let f be a mapping of X into Y where X and Y are metric spaces, then show that f is continuous at x_0 if and only if $x_n \longrightarrow x_0 \Longrightarrow f(x_n) \longrightarrow f(x_0)$.

14. Prove that every second countable space is separable but not conversely.

15. State and prove Lebesgue covering lemma.

16. State and prove Uryshon’s lemma.

17. a) Prove that the product of any non empty class of connected spaces is connected.

    b) Let A be a connected subspace of a topological space X and B is a subspace of X such that $A \subseteq B \subseteq \bar{A}$ then show that B is connected and in particular $\bar{A}$ is also connected.

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