STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2011-12)

SUBJECT CODE : 11MT/PC/LA24

M. Sc. DEGREE EXAMINATION, APRIL 2012
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : LINEAR ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

Section-A

Answer ALL the questions

(5x2=10)

1. Define a module and a Finitely generated module.
2. If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \alpha_2 T^2 + \cdots + \alpha_m T^m$, is invertible if $\alpha_0 \neq 0$.
3. Define characteristic value of a linear operator T on V and the characteristic value of A,where A is an nxn matrix over the field F.
4. Define adjoint of a linear operator. If T and U are linear operators on V, prove that $(TU)^* = U^*T^*$.
5. Let f be a form and T be a linear operator on finite-dimensional inner product space V. Prove that f is Hermition if and only if T is self adjoint.

Section-B

Answer any FIVE questions

(5x6=30)

6. Prove that intersection of any two submodules is a submodule.
7. Let $V = V_1 \oplus V_2$, where $V_1$and V_2 are subspaces of V invariant under T and let T_1 and T_2 be the linear transformation induced by T on $V_1$and V_2. Prove that minimal polynomial of T is the least common multiple of minimal polynomials of T_1 and T_2.
8. Let T be a linear operator on an n-dimensional vector space V. Prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

9. If T is a linear operator on a finite dimensional space V, prove that T is triangular if and only if the minimal polynomial for T is a product of linear polynomials over F.
10. Let V be a finite dimensional inner product space and f a linear functional on V. Then prove that there exists a unique vector β in V such that $f(\alpha) = (\alpha|\beta)$, for all $\alpha \; in \; V$.
11. Prove that, for any linear operator T on a finite dimensional inner product space V, there exists a unique linear operator T^* on V such that $(T\alpha|\,\beta) = (\,\alpha|T^*\beta)$, for all α, β in V.
12. Let V be a complex vector space and f a form on V such that $f(\alpha, \alpha)$ is real for every α. Then prove that f is Hermition.

Section-C

Answer any THREE questions

(3x20=60)

13. State and prove Fundamental Theorem on finitely generated modules.
14. State and prove Principal Axis theorem.
15. (a) State and prove Cayley Hamilton theorem.
    (b) Prove that minimal polynomial for $T \in A(V)$ divides the characteristic polynomial for $T \in A(V)$.
16. Prove that $S, T \in A(V)$ are similar if and only if they have same elementary divisors.
17. If T in $A(V)$ has an minimal polynomial $p(x) = q_1(x)^{e_1} q_2(x)^{e_2} \ldots q_k(x)^{e_k}$ over F, where $q_i(x)'s$ are distinct irreducible polynomial over F, obtain the rational canonical form of T.