STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**ELECTIVE**
PAPER	**:**	**DIFFERENTIAL GEOMETRY**
SUBJECT CODE	**:**	**23MT/PE/DG15**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

Q. No.	**SECTION A** $(\mathbf{5 \times 2 = 10})$ **Answer ALL questions**	**CO**	**KL**
1.	Prove that if the tangent vector of a parameterized curve is constant , the image of the curve is a straight line	1	1
2.	Define a Surface	1	1
3.	Define Developable	1	1
4.	What is Weingarten matrix of a surface patch?	1	1
5.	Define Umblic Point	1	1
Q. No.	**SECTION B** $(\mathbf{10 \times 1 = 10})$ **Answer ALL questions**	**CO**	**KL**
6.	If $\gamma(t)$ is a parameterized curve what will be its first derivative a) Tanget Vector b) Normal Vector c) Binormal Vector d) A scalar	2	2
7.	The Curvature of the helix is a) Vector b) Constant c) Derivative d) None of the above	2	2
8.	A smooth surface is a surface whose atlas consist of ------- patches a) Rough b) Smooth c) Rough and Smooth d) None of the above	2	2
9.	When does a point $\gamma(t)$ of parametrized curve γ is called regular point if ----------- a) Zero b) Approximately to zero c) Not equal to zero d) All of the above	2	2

10.	For a surface patches $\sigma: U \to R^3$ to be smooth then the vector product ------------ should be --------- at every point of U a) Partial Derivatives, Non-Zero b) Partial Derivatives, Zero c) First Derivative, one d) Second Derivative , two	2	2
11.	What is the second fundamental form of the plane is a) Non-zero b) Zero c) One d) Two	2	2
12.	All great circles on the sphere are a) Tangent Vector b) Normal Vector c) Geodesics d) Binormal Vector	2	2
13.	The Gaussian Curvature is the product of ------------------ curvatures a) Principal b) Ruled c) Product d) Sum	2	2
14.	A -----------is a surface generated by the motion of a straight line (called a generator) moving along a space curve. a) Ruled Surface b) Normal Surface c) Tangent Surface d) Binormal Sirface	2	2
15.	The Principal curvatures at a point of a surface are the maximum and minimum values of ----------- of all curves on the surface that passes through the point a) Principal Curvature b) Normal Curvature c) Tangent Curvature d) None of the above	2	2
Q. No.	**SECTION C ($\mathbf{2 \times 15 = 30}$)** **Answer ANY TWO questions**	**CO**	**KL**
16.	a) Prove that $\|\sigma_u \times \sigma_v\| = (EG - F^2)^{\frac{1}{2}}$ b) Compute the first fundamental form of $\sigma(u,v) = (u, v, u^2 + v^2)$ c) State and prove Serret – Frenet Equations	3	3

17.	a) Find the second fundamental form and the principal curvature for $\sigma(u,v) = (cosv, sinv, u)$ b) Compute the curvature and torsion of the circular helix.	3	3
18.	a) Prove that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere b) Prove: A diffeomorphism $f: S_1 \to S_2$ is an isometry if and only if for any surface patch σ_1 of S_1,the patches σ_1 and $f \circ \sigma_1$ of S_1 *and* S_2 respectively have the same first fundamental form.	3	3
19.	a) Derive the gaussian curvature, mean curvature and principal curvature in terms of the coefficients of first and second fundamental forms. b) Let κ_1 and κ_2 be the principal curvatures at a point P of a surface patch $\bar{\sigma}$. Then prove that (i) κ_1 and κ_2 are real numbers (ii) if $\kappa_1 = \kappa_2 = \kappa$, then $F_{II} = \kappa F_I$ and every tangent vector to $\bar{\sigma}$ at P is a principal vector: if $\kappa_1 \neq \kappa_2$, then any two (non-zero) principal vectors $\bar{t}_1$ and $\bar{t}_2$ corresponding to κ_1 and κ_2 , respectively are perpendicular	3	3
Q. No.	**SECTION D ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
20.	a) Prove that the Gaussian curvature of a ruled surface is negative or zero. b) Prove: A curve γ on a surface S is a geodesic if and only if, for any part $\gamma(t) = \sigma(u(t), v(t))$ of γ contained in a surface patch σ *of* S, then the geodesics equations are satisfied.	4	4
21.	a) Find the arc length and speed of logarithmic spiral. b) Let $\sigma(u,v)$ be a surface patch with first and second fundamental forms $Edu^2 + 2Fdudv + Gdv^2$ and $Ldu^2 + 2Mdudv + Ndv^2$ respectively. Then prove that the principal curvatures are $H \pm \sqrt{H^2 - K}$	4	4
22.	a) If $\gamma(t)$ is a regular curve in $\mathbb{R}^3$ then derive its curvature. b) Find the curvature of the curve $\left(\frac{4}{5}\cos t,\ 1 - \sin t,\ \frac{-3}{5}\cos t\right)$.	4	4
23.	a) Prove that the unit sphere in $\mathbb{R}^3$ is a surface. b) Prove that a parametrized curve has a unit-speed reparameterization if and only if it is regular.	4	4

Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	**CO**	**KL**
24.	State and prove the fundamental theorem on uniqueness and existence of space curves.	5	5
25.	a) State and prove Euler's theorem. b) Calculate the gaussian and mean curvature of the surface $\bar{\sigma}(u,v) = (u+v, u-v, uv)$ at the point (2, 0, 1).	5	5
26.	State and prove Gauss's remarkable theorem.	5	5
27.	a) Prove that $K = \frac{-\ddot{f}}{f}$ for the surface of revolution $\sigma(u,v) = (f(u)cosv, f(u)sinv, g(u))$ b) State and prove Meusnier's theorem	5	5

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