STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**ELECTIVE**
PAPER	**:**	**CALCULUS OF VARIATION AND INTEGRAL EQUATIONS**
SUBJECT CODE	**:**	**23MT/PE/CI15**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ALL questions	CO	KL
1.	Define linear functional.	1	1
2.	State reciprocity principle.	1	1
3.	Define Integral equation.	1	1
4.	When two continuous functions are said to be orthogonal?	1	1
5.	What is Dirac Delta function?	1	1
Q. No.	**SECTION B ($10 \times 1 = 10$) Answer ALL questions**	**CO**	**KL**
6.	In the classic Brachistochrone problem (finding the path of quickest descent between two fixed points under gravity), the functional to be minimized involves which of the following expressions for $F(y, y')$? $(a)\ \sqrt{1+y'^2}$ $\quad (b)\ \sqrt{\frac{1+y'^2}{2gy}}$ $(c)\ y\sqrt{1+y'^2}$ $\quad (d)\ \frac{1}{2}m\,y'^2 - mgy$	2	2
7.	To find the shortest distance between two fixed points (x_1, y_1) and (x_2, y_2) in a plane, we minimize the functional $I = \int_{x_1}^{x_2} \sqrt{1+(y')^2}dx$. The resulting extremal is (a) A parabola (b) A catenary (c) A straight line (d) A cycloid	2	2
8.	If a moving boundary point (x_b, y_b) must lie on a surface curve $y = \emptyset(x)$ the transversality condition at that point is given by $(a)\ \left[F + (y' - \emptyset')F_{y'}\right]_{x=x_b} = 0$ $(b)\ \left[F + (\emptyset' - y')F_{y'}\right]_{x=x_b} = 0$ $(c)\ \left[F + (y')F_{y'}\right]_{x=x_b} = 0$ $(d)\ \left[F_{y'}\right]_{x=x_b} = 0$	2	2

9.	In a variational problem where one or both boundary points are not fixed, the additional conditions required to determine arbitrary constants are known as (a) initial conditions (b) orthogonality conditions (c) Dirichlet conditions (d) Transversality conditions	2	2
10.	The integral equation $u(x) = \lambda \int_0^x K(x,t)u(t)dt$ is called (a) Fredholm equation of the first kind (b) Fredholem equation of the second kind (c) Volterra equation of the first kind (d) Volterra equation of the second kind	2	2
11.	A Fredholm equation is said to be homogeneous if (a) The kernel $K(x,t)$ is zero (b) The limits are the same (c) The function $f(x)$ outside the integral is zero (d) The unknown function $u(x)$ is zero	2	2
12.	Which of the following pairs of functions are orthogonal on the interval $[-\pi,\pi]$? $(a)\ f(x) = sin\, x\ g(x) = cos\, x$ $(b)\ f(x) = x\ g(x) = x$ $(c)\ f(x) = 1\ \ g(x) = x^2$ $(d)\ f(x) = e^x\ \ g(x) = 1$	2	2
13.	Let $K(x,t)$ be a symmetric kernel $\left(K(x,t) = K(t,x)\right)$ of a Fredhelom integral equation. If $\phi_1(x), \phi_2(x)$ are Eigen functions corresponding to distinct Eigen values λ_1 and λ_2 then they are _______ (a) linearly dependent (b) parallel (c) identical (d) orthogonal	2	2
14.	The shifting property of Dirac delta function: $\int_{-\infty}^{\infty} \emptyset(x)\delta(x)$ is _____, where $\emptyset(x)$ is continuous. $(a)\ \emptyset(0)$ $(b)\ \emptyset(1)$ $(c)\ \emptyset(x)$ (d) none of these	2	2
15.	Heaviside function $H(x)$ is _____ $(a)\ H(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$ $(b)\ H(x) = \begin{cases} 1, & x < 0 \\ 0, & x > 0 \end{cases}$ $(c)\ H(x) = \begin{cases} 0, & x < 0 \\ \infty, & x > 0 \end{cases}$ $(d)\ H(x) = \begin{cases} \infty, & x < 0 \\ 0, & x > 0 \end{cases}$	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
16.	Derive Euler's equation.	3	3
17.	Prove that $u(x) = cos2x$ is a solution of the integral equation $u(x) = cosx + 3\int_0^x K(x,t)u(t)dt$ where $K(x,t) = \begin{cases} sin\ x\ cos\ t, 0 \leq x \leq t \\ cos\ x\ sin\ t, t \leq x \leq \pi \end{cases}$	3	3
18.	Solve the following homogeneous Fredholm integral equation of the second kind $g(s) = \lambda \int_0^{2\pi} \sin(s+t)\, g(t)dt$	3	3
19.	Reduce the following boundary value problem to Fredholm integral equation $y'' + \lambda s y = 1, y(0) = 0, y(l) = 1$	3	3
Q. No.	**SECTION D** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
20.	Obtain the surface of minimum area stretched over a given closed curve and enclosing the domain D in the xy-plane.	4	4
21.	Test of an extremum the functional $\int_0^{x_1} \frac{\sqrt{1+y'^2}}{y} dx$ given that $y(0) = 0, y_1 = x_1 - 5$	4	4
22.	Reduce the Bessel equation $s^2 \frac{d^2y}{ds^2} + s\frac{dy}{ds} + (\lambda s^2 - 1)y = 0$ with end conditions $y(0) = 1, y(1) = 0$ to a Fredholm integral equation.	4	4
23.	Solve the inhomogeneous Fredholm integral equation of the second kind $y(x) = 2x + \lambda \int_0^1 (x+t)y(t)dt$ by the method of successive approximations to the third order taking $y_0(x) = 1$	4	4
Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	**CO**	**KL**
24.	Determine the extremal of the functional $v[y(x)] = \int_{-l}^{l} (\frac{1}{2}\mu y''^2 + \rho y)dx$, that satisfies the boundary conditions $y(-l) = y'(-l) = y(l) = y'(l) = 0$	5	5
25.	Determine the Eigen value and Eigen function of the homogeneous integral equation $g(x) = \lambda \int_0^1 e^{x+t} g(t)dt$	5	5

26.	Determine the resolvent kernel for the Fredholm integral equation having kernel $K(x,t) = e^{x+t}; a = 0, b = 1$	5	5
27.	Determine the transversality condition for the functional $v = \int_{x_0}^{x_1} A(x,y,z)\sqrt{1+y'^2+z'^2}dx \quad if\ z_1 = \varphi(x_1, y_1)$	5	5

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