STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
SECOND SEMESTER

COURSE	**:**	**CORE**	
PAPER	**:**	**TOPOLOGY**	
SUBJECT CODE	**:**	**23MT/PC/TO24**	
TIME	**:**	**3 HOURS**	**MAX. MARKS: 100**

Q. No.	SECTION A $(5 \times 2 = 10)$ Answer ALL questions	CO	KL
1.	Define a topological space.	1	1
2.	Define a metric on a set S.	1	1
3.	When does a space X is said to be locally connected at a point 'x'.	1	1
4.	Define a compact space.	1	1
5.	Define Regular and normal spaces.	1	1
Q. No.	**SECTION B $(10 \times 1 = 10)$** **Answer ALL questions**	**CO**	**KL**
6.	Given a set X define $d(x, y) = 1$ if $x \neq y$ $d(x, y) = 0$ if $x = y$ The topology it induces is ----------- a) Discrete topology b) Product topology c) Box topology d) Order topology	2	2
7.	If B is the collection of all open intervals in the real line $(a, b) = \{x \mid a < x < b\}$ then the topology generated by B on the rea line is called ----------------. a) Standard topology b) Product topology c) Box topology d) Order topology	2	2
8.	The uniform topology on the cartesian product $\mathfrak{R}^J$ is finer than the ------------- a) Standard topology b) Product topology c) Box topology d) Order topology	2	2
9.	If $S_\beta = \{\pi_\beta^{-1}(U_\beta) \mid U_\beta$ open in $X_\beta\}$and if S is the union of these collections $S = \cup S_\beta$, $\beta \in J$, J is an index set, then the topology generated by the subbasis S is called --------- a) Standard topology b) Product topology c) Box topology d) Order topology	2	2

10.	If $f : [a, b] \to R$ is continuous and if r is a real number between $f(a)$ and $f(b)$, then there exists an element $c \in [a, b]$ such that $f(c) = r$. This theorem is called as ----------- a) Uniform continuity theorem b) Minimum value theorem c) Maximum value theorem d) Intermediate value theorem.	2	2
11.	If X is a two-point space in the indiscrete topology then X is ___ a) Connected b) Disconnected c) Finite d) None	2	2
12.	A space X is said to be limit point compact if ------------ subset of X has a limit point. a) every infinite b) every finite c) every d) None of the above	2	2
13.	Every closed interval in $\mathbb{R}$ is-----------------. a) Uncountable b) finite c) enumerable d) None	2	2
14.	If a space X has a countable basis for its topology, then X is said to be --------------------. a) Uncountable b) countable c) enumerable d) second countable	2	2
15.	Every regular space with a countable basis is --------------------. a) normal b) regular c) Hausdorff d) Lindelof	2	2
Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
16.	Show that if X and Y are topological spaces and $f: X \longrightarrow Y$ then the following are equivalent. (i) f is continuous (ii) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$. (iii) For every closed subset B of Y, the set $f^{-1}(B)$ is closed in X. (iv) For each $x \in X$ and each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset V$.	3	3
17.	Show that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.	3	3

18.	If L is a linear continuum in the order topology, prove that L is connected and so are the intervals and the rays on L.	3	3
19.	Prove that if X, a nonempty compact Hausdorff space, has no isolated points then it is uncountable.	3	3
Q. No.	**SECTION D** $(\mathbf{2\times 15=30})$ **Answer ANY TWO questions**		
20.	If X is a metric space with 'd' and $\bar{d}$: $X \times X \longrightarrow R$ is defined by the equation $\bar{d}(x,y) = \min\{d(x,y),1\}$ then show that $\bar{d}$ is a metric that induces the same topology as 'd'.	4	4
21.	If X is a simply ordered set having the least upper bound property, prove that each closed interval in X is compact in the order topology.	4	4
22.	If X is a metrizable space, prove that the following are equivalent: (i) X is compact. (ii) X is limit point compact. (iii) X is sequentially compact.	4	4
23.	State and prove Urysohn Lemma.	4	4
Q. No.	**SECTION E** $(\mathbf{2\times 10=20})$ **Answer ANY TWO questions**	**CO**	**KL**
24.	If X is a set and $\mathcal{B}$ is a basis for a topology $\mathfrak{I}$ on X, then prove that $\mathfrak{I}$ equals the collection of all unions of elements of $\mathcal{B}$.	5	5
25.	Prove that the image of a connected space under a continuous map is connected.	5	5
26.	Prove that a subspace of a completely regular space is completely regular and a product of completely regular spaces is completely regular.	5	5
27.	Prove that every compact Hausdorff space is normal.	5	5

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