STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**STOCHASTIC PROCESSES**
SUBJECT CODE	**:**	**23MT/PC/SP44**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ALL questions	CO	KL
1.	Define counting process.	1	1
2.	Define *n*-step transition probabilities.	1	1
3.	When we say that the chain is ergodic.	1	1
4.	Let $X_1, X_2, \ldots$ be independent random variables with 0 mean; and let $Z_n = \sum_{i=1}^{n} X_i$. Then prove that $\{Z_n, n \geq 1\}$ is a martingale.	1	1
5.	Define Gaussian processes.	1	1
Q. No.	**SECTION B ($10 \times 1 = 10$) Answer ALL questions**	**CO**	**KL**
6	In a counting process $N(t)$ must satisfy, for $s < t$, $N(t) - N(s)$ equals the number of events that occurred in the interval is ______________ (A) (s,t) (B) $[s,t]$ (C) $[s,t)$ (D) $(s,t]$	2	2
7.	$X_n, n = 1, 2, \ldots$ are independent identically distributed exponential random variables having mean ______________ (A) λ (B) λ^2 (C) $\frac{1}{\lambda}$ (D) $\frac{1}{\lambda^2}$	2	2
8.	State *j* is said to be accessible from state *i* if for some ____ (A) $n > 0,\ P_{ij}^n \geq 0$ (B) $n > 0,\ P_{ij}^n > 0$ (C) $n \geq 0,\ P_{ij}^n \geq 0$ (D) $n \geq 0,\ P_{ij}^n > 0$	2	2
9.	A probability distribution $\{P_j, j \geq 0\}$ is said to be stationary for the Markov chain if__________ (A) $P_j = \sum_{i=0}^{\infty} P_i P_j$ (B) $P_i = \sum_{i=0}^{\infty} P_i P_{ij}$ (C) $P_j = \sum_{i=0}^{\infty} P_i P_{ij}$ (D) $P_i = \sum_{j=0}^{\infty} P_j P_{ij}$	2	2

10.	A continuous-time Markov chain is said to be regular if, __________ (A) with probability 1, the number of transitions in any infinite length of time is finite. (B) with probability 0, the number of transitions in any infinite length of time is infinite. (C) with probability 1, the number of transitions in any finite length of time is finite. (D) with probability 1, the number of transitions in any finite length of time is infinite	2	2
11.	A continuous-time Markov chain with states 0,1,… for which $q_{ij} = 0$ whenever ________ is called a birth and death process. (A) $\|i-j\| \geq 1$ (B) $\|i-j\| > 1$ (C) $\|i-j\| < 1$ (D) $\|i-j\| \leq 1$	2	2
12.	Let $X_{1,} X_2,...$ be independent random variables with 0 mean and let $Z_n = \sum_{i=1}^{n} X_i$, then _______ (A) $E\left[Z_{n+1} \mid Z_1,...,Z_n\right] = Z_n$ (B) $E\left[Z_{n+1} \mid Z_1,...,Z_n\right] = Z_{n+1}$ (C) $E\left[Z_{n+1} \mid Z_1,...,Z_{n+1}\right] = Z_n$ (D) $E\left[Z_n \mid Z_1,...,Z_n\right] = Z_n$	2	2
13	If $\{Z_i, i \geq 1\}$ is a submartingale and N a stopping time such that ________, then $E[Z_1] \leq E[Z_N] \leq E[Z_n]$. (A) $P\{N > n\} = 1$ (B) $P\{N \geq n\} = 1$ (C) $P\{N \leq n\} = 1$ (D) $P\{N < n\} = 1$	2	2
14.	A stochastic process $\left[X(t), t \geq 0\right]$ is said to be Brownian motion process if. (A) for every $t > 0$, $X(t)$ is normally distributed with mean 0 and variance c^2. (B) for every $t \geq 0$, $X(t)$ is normally distributed with mean 0 and variance c^2. (C) for every $t > 0$, $X(t)$ is normally distributed with mean 0 and variance $c^2 t$. (D) for every $t \geq 0$, $X(t)$ is normally distributed with mean 0 and variance $c^2 t$.	2	2

15.	The Brownian Bridge can be defined as a Gaussian process with mean 0 and ____________ (A) covariance function $s(1-t),\ s<t$. (B) covariance function $s(1-t),\ s\le t$. (C) covariance function $s(t-1),\ s<t$. (D) covariance function $s(t-1),\ s\le t$.	2	2
Q. No.	**SECTION C** ($\mathbf{2\times 15=30}$) **Answer ANY TWO questions**	**CO**	**KL**
16.	Let $\tau_1,\tau_2,\dots,\tau_n$ denote the ordered values from a set of n independent uniform $(0,t)$ random variables. Let $Y_1,Y_2,$ be independent and identically distributed nonnegative random variables are also independent of $\{\tau_1,\tau_2,\dots,\tau_n\}$ Then prove that $P\{Y_1+\cdots+Y_k<\tau_k, k=1,2,\dots,n \mid Y_1+\cdots+Y_n=y\}$ $=\begin{cases}1-\frac{y}{t} & 0<y<t\\ 0 & \text{otherwise}\end{cases}$.	3	3
17.	(i) Prove that communication is an equivalence relation. (ii) If $i\leftrightarrow j$, then prove that $d(i)=d(j)$. (8+7)	3	3
18.	Consider a Yule process with $X(0)=1$. Compute the expected sum of the ages of the members of the population at time *t*.	3	3
19.	If N is a random time for the martingale $\{Z_n\}$, then prove the stopped process $\{\bar{Z}_n\}$ is also a martingale.	3	3
Q. No.	**SECTION D** ($\mathbf{2\times 15=30}$) **Answer ANY TWO questions**	**CO**	**KL**
20.	Prove that an irreducible aperiodic Markov chain belongs to one of the two classes: (i) Either the states are all transient or all null recurrent; in this case, $P_{ij}^n\to 0$ as $n\to\infty$, for all i,j and there exists no stationary distribution. (ii) or else, all states are positive recurrent, i.e., $\pi_j=\lim_{n\to\infty}P_{ij}^n>0$. In this case, $\{\pi_j, j=0,1,2,\dots\}$ is a stationary distribution and there exist no other stationary distribution.	4	4

21.	Discuss two birth and death processes.	4	4
22.	Players *X*, *Y* and *Z* contest the following game. At each stage two of them are randomly chosen in sequence, with the first one chosen being required to give 1 coin to the other. All of the possible choices are equally likely and successive choices are independent of the past. This continues until one of the players has no remaining coins. At this point that player departs and the other two continue playing until one of them has all the coins. If the players initially have *x*, *y* and *z* coins, find the expected number of plays until one of them has all the $s \equiv x+y+z$ coins.	4	4
23.	Discuss Brownian motion absorbed at a value.	4	4
Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	**CO**	**KL**
24.	Given that $\mathbf{N}(t)=n$, prove that the n arrival times $S_1, S_2, \ldots, S_n$ have the same distribution as the order statistics corresponding to n independent random variables uniformly distributed on the interval $(0,t)$.	5	5
25.	Prove that state *j* is recurrent, if and only if $\sum_{n=1}^{\infty} P_{jj}^n = \infty$.	5	5
26.	Discuss a simple Epidemic model.	5	5
27.	Discuss Integrated Brownian motion.	5	5