**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86**
**(For candidates admitted from the academic year 2023 – 2024 and thereafter)**

**M.Sc. DEGREE EXAMINATION, APRIL 2026**
**BRANCH I - MATHEMATICS**
**SECOND SEMESTER**

| | | | |
|---|---|---|---|
| **COURSE** | **:** | **CORE** | |
| **PAPER** | **:** | **LINEAR ALGEBRA** | |
| **SUBJECT CODE** | **:** | **23MT/PC/LA24** | |
| **TIME** | **:** | **3 HOURS** | **MAX. MARKS: 100** |

| **Q. No.** | **SECTION A** $(\mathbf{5 \times 2 = 10})$<br>**Answer ALL questions** | **CO** | **KL** |
|---|---|---|---|
| 1. | When do we say that two linear transformations are similar? | 1 | 1 |
| 2. | Define basic Jordan block belonging to $\lambda$ and Jordan form of a transformation $T$. | 1 | 1 |
| 3. | What is meant by an invariant subspace? | 1 | 1 |
| 4. | Show that the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is orthogonal | 1 | 1 |
| 5. | Define positive matrix | 1 | 1 |
| **Q. No.** | **SECTION B** $(\mathbf{10 \times 1 = 10})$<br>**Answer ALL questions** | **CO** | **KL** |
| 6. | If $A$ is triangular and the elements on its main diagonal are 0, then<br>(a) $\lvert A\rvert \neq 0$ (b) all its eigen values are non-zero<br>(c) $A$ is nilpotent (d) none of these | 2 | 2 |
| 7. | If $dim\ V = n$, over F and if $T \in A(V)$ has all its characteristic roots in $F$, then $T$ satisfies a polynomial of degree<br>(a) 1 (b) 0 (c) $n$ (d) $n+1$ | 2 | 2 |
| 8. | If $dim\ V = n$, then the characteristic polynomial of $T$, $p_T(x)$ is<br>(a) sum of its elementary divisors<br>(b) product of its elementary divisors<br>(c) polynomial of degree $> n$<br>(d) none of the above | 2 | 2 |
| 9. | The rational canonical form is based on:<br>(a) Companion matrices of invariant factors<br>(b) Diagonalization of symmetric matrices<br>(c) Orthogonalization of eigenvectors<br>(d) Singular value decomposition | 2 | 2 |

| | | | |
|---|---|---|---|
| 10. | If $T$ has characteristic value $c$, then the operator $T - cI$ is<br>(a) Invertible (b) the zero operator<br>(c) Singular (d) diagonal | 2 | 2 |
| 11. | A polynomial $p$ is said to annihilate $T$ if<br>(a) $p(T) = I$ (b) $p(T) = 0$ (c) $p(T) = T$ (d) $p(T) = T^{-1}$ | 2 | 2 |
| 12. | A matrix representing a unitary operator in an orthonormal basis satisfies<br>(a) $AA^* = I$ (b) $A^2 = I$ (c) $A = A^T$ (d) $A = 0$ | 2 | 2 |
| 13. | A linear operator $T$ is normal if<br>(a) $TT^* = T^*T$ (b) $T = T^*$<br>(c) $T^2 = I$ (d) $T$ is diagonalizable | 2 | 2 |
| 14. | A function $f$ on an inner product space $V$ is called a form if<br>(a) $f: V \to F$ is linear (b) $f: V \times V \to F$<br>(c ) $f: F \to V$ (d) $f: V \to V$ | 2 | 2 |
| 15. | A form $f$ is called positive if<br>(a) $f$ is Hermitian & $f(x, x) < 0$ for all $x$ (b) $f(x, y) = 0$<br>(c ) $f$ is Hermitian & $f(x, x) \geq 0$ for all $x$ (d) $f(x, y) = 1$ | 2 | 2 |
| **Q. No.** | **SECTION C ($2 \times 15 = 30$)**<br>**Answer ANY TWO questions** | **CO** | **KL** |
| 16. | If $T \in A(V)$ has all its characteristic roots in $F$, then show that there is a basis of $V$ in which the matrix of $T$ is triangular | 3 | 3 |
| 17. | Let $T \in A(V)$ and suppose that $p(x) \in F[x]$ is the minimal polynomial of $T$ and $p(x) = q_1(x)^{l_1} q_2(x)^{l_2} \ldots q_k(x)^{l_k}$ where the $q_i(x)$ are distinct irreducible polynomials and $l_1, l_2, \cdots, l_k$ are positive integers. If $V_i = \{ v \in V \mid v q_i(T)^{l_i} = 0\}$, prove that each $V_i$ is invariant under $T$, $V_i \neq (0)$ and $V = V_1 \oplus V_2 \cdots \oplus V_k$. Also prove that the minimal polynomial of $T_i$ is $q_i(x)^{l_i}$ | 3 | 3 |
| 18. | State and prove Cayley Hamilton theorem | 3 | 3 |
| 19. | Let $V$ be a finite-dimensional inner product space and $f$ a form on $V$. Prove that there is a unique linear operator $T$ on $V$ such that $f(\alpha, \beta) = (T\alpha \vert \beta)$ for all $\alpha, \beta$ in $V$ and the map $f \to T$ is an isomorphism of the space of forms onto $L(V, V)$. | 3 | 3 |

| Q. No. | **SECTION D** $(\mathbf{2 \times 15 = 30})$ <br> **Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 20. | If $T \in A(V)$ is nilpotent, of index of nilpotence $n_1$, then show that a basis of $V$ can be found such that the matrix of $T$ in this basis has the form $\begin{pmatrix} M_{n_1} & 0 & \cdots & 0 \\ 0 & M_{n_2} & \cdots & 0 \\ & \vdots & & \\ \cdots & & & M_{n_r} \end{pmatrix}$ where $n_1 \geq n_2 \geq \cdots \geq n_r$ and $n_1 + n_2 + \cdots + n_r = dim\, V$. | 4 | 4 |
| 21. | Prove that for every invertible complex $n \times n$ matrix $B$ there exists a unique lower-triangular matrix $M$ with positive entries on the main diagonal such that $MB$ is unitary. | 4 | 4 |
| 22. | Let $V$ be a finite-dimensional inner product space, and let $T$ be a self-adjoint linear operator on $V$, then prove that there is an orthonormal basis for $V$ each vector of which is a characteristic vector for $T$. | 4 | 4 |
| 23. | Let $A$ be an invertible $n \times n$ matrix with entries in a field $F$. Then prove the following statements are equivalent. <br> (a) There is an upper-triangular matrix $P$ with $P_{kk} = 1$ $(1 \leq k \leq n)$ such that the matrix $B = AP$ is lower-triangular. <br> (b) The principal minors of $A$ are all different from 0. | 4 | 4 |
| **Q. No.** | **SECTION E** $(\mathbf{2 \times 10 = 20})$ <br> **Answer ANY TWO questions** | **CO** | **KL** |
| 24. | If $V$ is $n$-dimensional over $F$ and if $T \in A(V)$ has all its characteristic roots in $F$, then show that $T$ satisfies a polynomial of degree $n$ over $F$. | 5 | 5 |
| 25. | Make use of similarity, prove that the elements $S, T \in A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors. | 5 | 5 |
| 26. | Let $T$ be a linear operator on $n$-dimensional vector space $V$. Prove that the characteristic and minimal polynomials for $T$ have the same roots, except for multiplicities. | 5 | 5 |
| 27. | State and prove principal axis theorem | 5 | 5 |