STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

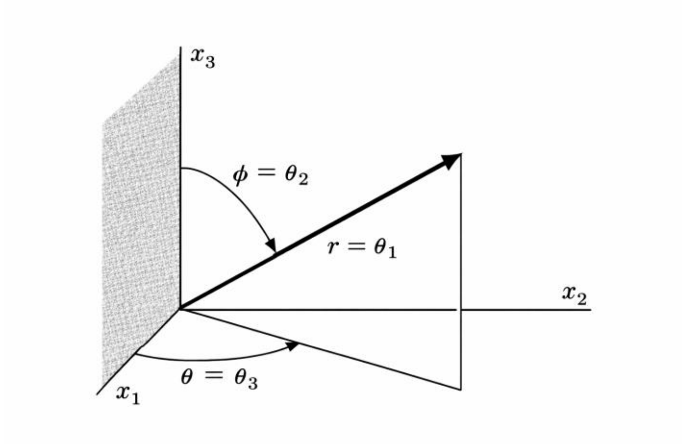
M.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**CONTINUUM AND FLUID MECHANICS**
SUBJECT CODE	**:**	**23MT/PC/CF44**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ALL questions	CO	KL
1.	Define the conjugate and scalar of a dyadic **D**.	1	1
2.	Write the Cauchy's deformation tensor.	1	1
3.	Define vorticity vector and vortex line.	1	1
4.	State Pascal's law.	1	1
5.	Describe the no-slip condition.	1	1
Q. No.	**SECTION B ($10 \times 1 = 10$) Answer ALL questions**	**CO**	**KL**
6.	The dyadic product ab is called a nonion because a) It represents a scalar quantity b) It has three components c) It has nine components in three-dimensional space d) It represents a vector quantity	2	2
7.	A second-order tensor D_{ij} is called antisymmetric if a) $D_{ij} = D_{ji}$ b) $D_{ij} = -D_{ji}$ c) $D_{ij} = 0$ d) $D_{ij} = \delta_{ji}$	2	2
8.	The stress vector acting on a surface is very often referred to as the a) Force vector b) Velocity vector c) Strain vector d) Traction vector	2	2
9.	The difference $(dx)^2 - (dX)^2$ for two neighboring particles of a continuum is used as a measure of a) Deformation between the initial and final configurations b) Rigid body motion c) Rotation of particles d) Translation of the continuum	2	2

10.	In a fluid flow, the necessary and sufficient condition for the existence of a velocity potential ϕ is that a) $\nabla \bullet \vec{q} = 0$ b) ϕ = constant everywhere c) $\nabla \times \vec{q} = 0$ d) Streamlines are parallel	2	2
11.	A point in a fluid from which the flow is directed radially outward in all directions in a symmetrical manner is called a a) Simple source b) Simple sink c) Doublet d) Vortex	2	2
12.	At any point in a moving inviscid fluid, the pressure is a) Dependent on the choice of axes b) Greater in the direction of flow c) Zero in the transverse direction d) The same in all directions	2	2
13.	For an irrotational incompressible two-dimensional flow, the velocity potential ϕ and stream function ψ are a) Constant everywhere in the flow field b) Harmonic functions and their curves intersect orthogonally c) Linearly dependent functions d) Independent and non-orthogonal	2	2
14.	According to Newton's law of viscosity, the shear stress τ is given by a) $\tau = \rho \frac{du}{dy}$ b) $\tau = \mu \frac{dp}{dx}$ c) $\tau = \mu \frac{du}{dy}$ d) $\tau = \frac{du}{dt}$	2	2
15.	The Navier–Stokes equation reduces to Euler's equation when a) Velocity is zero b) Viscosity is neglected c) Pressure is constant d) Flow is steady	2	2

Q. No.	SECTION C ($2 \times 15 = 30$) **Answer ANY TWO questions**	CO	KL
16.	(a) For the vector $\vec{a} = 3\hat{i} + 4k$, $\vec{b} = 2\hat{i} - 6k$ and the dyadic $D = 3\hat{\imath}\hat{\imath} + 2\hat{\imath}\hat{k} - 4\hat{\jmath}\hat{\jmath} - 5\hat{k}\hat{\jmath}$, Compute by matrix multiplication the products of $\vec{a} \bullet D$, $D \bullet \vec{b}$ and $\vec{a} \bullet D \bullet \vec{b}$ (8) (b) Prove that (i) $\left(D_c \bullet D\right)_c = D_c \bullet D$ (ii) $D \bullet \vec{v} = \vec{v} \bullet D_c$ (7)	3	3
17.	(a) Discuss the stress transformation laws. (10) (b) The Lagrangian description of a deformation is given by $x_1 = X_1 + X_3\left(e^2 - 1\right)$, $x_2 = X_2 + X_3\left(e^2 - e^{-2}\right)$, $x_3 = e^2 X_3$, where *e* is a constant. Show that the Jacobian *J* does not vanish and determine the Eulerian equations describing this motion. (5)	3	3
18.	(a) Discuss the flow for which $w = z^2$. (5) (b) Doublets of strengths μ_1, μ_2 are situated at points A_1, A_2 whose Cartesian coordinates are $(0,0,c_1)$, $(0,0,c_2)$, their axes being directed towards and away from the origin respectively. Find the condition that there is no transport of fluid over the surface of the sphere $x^2 + y^2 + z^2 = c_1 c_2$ (10)	3	3
19.	Discuss steady viscous flow in a tube having a uniform elliptical cross-section.	3	3
Q. No.	**SECTION D** ($2 \times 15 = 30$) **Answer ANY TWO questions**	**CO**	**KL**
20.	(a) For the permutation symbol $\in_{ijk}$ show by direct expansion that (i) $\in_{ijk} \in_{kij} = 6$ (ii) $\in_{ijk}\, a_j\, a_k = 0$ (8) (b) Resolve the dyadic $D = 3\hat{\imath}\hat{\imath} + 4\hat{\imath}\hat{k} + 6\hat{\jmath}\hat{\imath} + 7\hat{\jmath}\hat{\jmath} + 10\hat{k}\hat{\imath} + 2\hat{k}\hat{\jmath}$ into its symmetric and antisymmetric parts. (7)	4	4
21	(a) Derive the relationship between the stress tensor and the stress vector. (10) (b) A continuum body undergoes the deformation $x_1 = X_1$, $x_2 = X_2 + AX_3$, $x_3 = X_3 + AX_2$ where A is a constant. Compute the deformation tensor **G** and use this to determine the Lagrangian finite strain tensor $\mathbf{L_G}$. (5)	4	4

22.	At any point in an incompressible fluid having spherical polar coordinates (r,θ,ψ), the velocity components are $\left[2M\,r^{-3}\sin\left(\frac{\pi}{2}-\theta\right), M\,r^{-3}\cos\left(\frac{\pi}{2}-\theta\right), 0\right]$, where M is a constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of streamlines.	4	4
23.	(a) Derive the Euler's equation of motion. (10) (b) State and prove uniqueness theorem. (5)	4	4
Q. No.	**SECTION E ($2 \times 10 = 20$)** **Answer ANY TWO questions**	**CO**	**KL**
24.	Determine directly the components of the metric tensor for spherical polar coordinates as shown in the figure. 	5	5
25.	The stress tensor values at P are given by the array $\sum = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$. Determine the traction (stress) vector on the plane at P whose unit normal is $n = \left(\frac{2}{3}\right)\hat{e}_1 - \left(\frac{2}{3}\right)\hat{e}_2 + \left(\frac{1}{3}\right)\hat{e}_3$ and for the traction vector find (i) the component perpendicular to the plane, (ii) the magnitude of $t_i^{(n)}$ (iii) the angle between $t_i^{(n)}$ and n.	5	5
26.	(a) Obtain the steady-state form of the continuity equation for an incompressible fluid. (5) (b) Derive the Bernoulli's equation for a homogeneous and incompressible fluid. (5)	5	5
27.	Derive the Navier- Stokes equation of motion of a viscous fluid.	5	5

⅄⅄⅄⅄⅄⅄⅄