STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**	
PAPER	**:**	**COMPLEX ANALYSIS**	
SUBJECT CODE	**:**	**23MT/PC/CA44**	
TIME	**:**	**3 HOURS**	**MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ALL questions	CO	KL
1.	Show that $n(\gamma, a) = 0, \forall a$ outside of a circle and γ lies inside of the same circle.	1	1
2.	Define the term: Homologous to zero	1	1
3.	Write Legendre's Duplication formula.	1	1
4.	State Riemann Mapping theorem	1	1
5.	State the Mean value Property of a function in a domain.	1	1
Q. No.	**SECTION B ($10 \times 1 = 10$) Answer ALL questions**	**CO**	**KL**
6.	The value of $\int_\gamma e^{\frac{1}{z}} dz$ where $\gamma: \lvert z \rvert = 1$is (a)1 (b)-1 (c) 0 (d) 2	2	2
7.	If a curve $\gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$, winds twice counterclockwise and once clockwise around a, then the winding number is (a)1 (b) 0 (c) -1 (d) depends on r	2	2
8.	Which of the following statements is false? (a) A harmonic function satisfies the mean value property (b) Harmonicity is preserved under uniform limits (c) The real part of an analytic function is harmonic (d) A harmonic function can have a strict interior maximum	2	2
9.	The derivatives f_n' converge uniformly on compact subsets provided (a) f_n converge pointwise (b) f_n are harmonic (c) f_n are bounded at one point (d) f_n converge uniformly on compact subsets	2	2

10.	Let $\mathcal{F}$be a family of analytic functions uniformly bounded on compact subsets of a domain Ω. Then $\mathcal{F}$ is (a) uniformly convergent (b) Equi continuous on compact subsets (c) compact in Ω (d) pointwise convergent	2	2
11.	Uniform convergence of an infinite product on compact sets ensures that the limit function is (a) analytic (b) bounded (c) harmonic (d)injective	2	2
12.	A family $\Im$ is normal iff its closure $\overline{\square}$with respect to the distance function is (a) normal (b) constant (c) compact (d) none of the above	2	2
13.	An infinite product $\prod_{n=1}^{\infty}(1+a_n)$ converges if and only if (a) $a_n \to 0$ (b) $\sum a_n$converges (c) $\log\sum(1+a_n)$converges (d) $\sum \mid a_n \mid$converges	2	2
14.	Which property of the boundary is essential for applying reflection arguments in conformal mapping? (a) Compactness (b) Boundedness (c) One-sidedness (d) Differentiability everywhere	2	2
15.	The Riemann Mapping Theorem fails for $\Omega = \mathbb{C}$ because: (a) $\mathbb{C}$ is not open (b) There is no bounded holomorphic function on $\mathbb{C}$ (c) $\mathbb{C}$ is not simply connected (d) Normal families do not exist	2	2
Q. No.	**SECTION C ($\mathbf{2 \times 15 = 30}$)** **Answer ANY TWO questions**	**CO**	**KL**
16.	State and prove the Cauchy's theorem for Rectangle.	3	3
17.	Prove that the function $P_{U(z)}$ is harmonic for $\lvert z\rvert < 1$ and $\lim\limits_{z\to e^{i\theta_0}} U(\theta_0)$ provided that U is continuous at θ_0.	3	3
18.	State and prove Mittag-Leffler theorem	3	3
19.	State and prove the Schwarz-Christoffel Formula for conformal Mapping of Polygons.	3	3

Q. No.	**SECTION D** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
20.	State and prove a General form of Cauchy's Theorem.	4	4
21.	Prove that $\zeta(s) = 2^s \pi^{s-1} \sin\frac{\pi s}{2} \Gamma(1-s)\zeta(1-s)$ ($\zeta(s)$ denotes usual notation of zeta function)	4	4
22.	State and prove Arzela's - Ascoli theorem.	4	4
23.	Discuss the Fluid flow Channel through slit, hence derive complex potential function $F(w)$ for the flow in the upper half of the z plane.	4	4
Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	**CO**	**KL**
24.	State and prove Cauchy's Integral formula	5	5
25.	Derive the Poisson's formula.	5	5
26.	Derive the Jensen's formula for entire functions.	5	5
27.	Prove the statement: Suppose that the boundary of a simply connected region Ω contains a line segment γ as a one-sided free boundary arc. Then the function $f(z)$ which maps Ω on to the unit disk can be extended to a function which is analytic and one to one on $\Omega \cup \gamma$.The image of γ is an arc γ' on the unit circle.	5	5

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