STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023– 2024)

B.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I - MATHEMATICS
SIXTH SEMESTER

COURSE	**:**	**MAJOR CORE**
PAPER	**:**	**VECTOR SPACES AND LINEAR TRANSFORMATIONS**
SUBJECT CODE	**:**	**23MT/MC/VL64**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ANY FIVE questions	CO	KL
1.	Define a vector space.	1	1
2.	What is meant by dilation of linear transformation.	1	1
3.	Define the kernel of a linear transformation.	1	1
4.	What is a coordinate vector of a vector relative to a basis?	1	1
5.	Define an inner product on a real vector space.	1	1
6.	State the Rank-Nullity Theorem.	1	1

Q. No.	SECTION B ($10 \times 1 = 10$) Answer ALL questions	CO	KL
7.	The set of all linear combinations of a given set of vectors is called (a) basis (b) dimension (c) span (d) rank	2	2
8.	The dimension of a vector space V is: (a) The number of elements in V (b) The number of vectors in any basis of V (c) The number of subspaces of V (d) The rank of any matrix representing V	2	2
9.	Which of the following statements is TRUE? (a) Every spanning set is linearly independent (b) Every linearly independent set spans the vector space (c) A basis is a linearly independent spanning set (d) A basis always contains the zero vector	2	2
10.	If a matrix A is diagonalizable, then: (a) A has n distinct eigenvalues (b) A has n linearly independent eigenvectors (c) A is symmetric (d) A is orthogonal	2	2

11.	If $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a linear transformation defined by $T(x) = Ax$, then T is one-to-one if and only if: (a) A is singular (b) A is nonsingular (c) rank(A) < n (d) nullity(A) = n	2	2
12.	The projection of a vector v onto a subspace W gives: (a) A vector orthogonal to W (b) The closest point in W to v (c) A basis for W (d) The norm of v	2	2
13.	The range of a linear transformation is a (a) scalar (b) subspace (c) basis (d) matrix	2	2
14.	Two vectors u and v in an inner product space are orthogonal if: (a) $\|u\| = \|v\|$ (b) $< u, v > = 0$ (c) $u + v = 0$ (d) $< u, u > \geq < v, v >$	2	2
15.	The norm of a vector is (a) a vector (b) a scalar (c) a matrix (d) a transformation	2	2
16.	The least squares solution of $Ax = y$ is used when: (a) The system has a unique solution (b) The system has infinitely many solutions (c) The system has no solution (d) The system has finite number of solutions	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
17.	(a) Show that the set P_2 of all polynomials of degree at most 2, with the usual addition and scalar multiplication, forms a vector space. (b) Determine whether the set of vectors {(1, 2, 3), (2, 4, 6), (1, 0, 1)} in $\mathbb{R}^3$ is linearly independent or dependent. Justify your answer. (7+8)	3	3

18.	(a) Find the projection of the vector $v = (6, 7)$ onto the vector $u = (1, 4)$ in $\mathbb{R}^2$. (b) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (x + y, 2z)$. Find the kernel and range of T. Verify the Rank-Nullity theorem for this transformation. (7+8)	3	3
19.	(a) Let B and B' be bases for a vector space U and P be the transition matrix from B to B', then prove that P is invertible and the transition matrix from B' to B is P^{-1}. (b) Let $B = \{(1,2), (3, -1)\}$ and $B' = \{(3,1), (5,2)\}$ in $\mathbb{R}^2$. Find the transition matrix from B to B'. If u is a vector such that $u_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then find $u_{B'}$. (8+7)	3	3
20.	Find the least squares line $y = a + bx$ that best fits the data points: $(1, 1), (2, 2.4), (3, 3.6), (4, 4)$.	3	3

Q. No.	**SECTION D** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
21.	a) Let the vectors $\{v_1, v_2, \ldots, v_n\}$ span a vector space V. Prove that each vector in V can be expressed uniquely as a linear combination of these vectors if and only if the vectors are linearly independent. (b) Prove that the nonzero row vectors of a matrix A that is in reduced echelon form are a basis for the row space of A. The rank of A is the number of nonzero row vectors. (8+7)	4	4
22.	(a) State and prove Gram–Schmidt orthogonalization process to obtain an orthonormal basis for a given vector space. (b) The set $\{(1,2,0,3), (4,0,5,8), (8,1,5,6)\}$ is linearly independent in The vectors form a basis for a three-dimensional subspace V of Construct an orthonormal basis for V. (6+9)	4	4
23.	Prove that an orthogonal transformation preserves norms, angles, and distances.	4	4
24.	Let A be an $n \times n$ symmetric matrix. Then prove that (a) If A is real, all the eigenvalues of A are real numbers. (b) The dimension of an eigenspace of A is the multiplicity of the eigenvalue as a root of the characteristic equation. (c) The eigenspaces of A are orthogonal	4	4

Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	**CO**	**KL**
25.	Evaluate whether the set $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 1\}$ is a subspace of $\mathbb{R}^3$. Justify your conclusion and compare to the set $V = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$?	5	5
26.	State and prove orthogonal matrix theorem.	5	5
27.	Let T be a linear transformation of $\mathbb{R}^n \to \mathbb{R}^n$, then prove that T is invertible if and only if it is nonsingular. Also prove that the inverse is unique and linear.	5	5
28.	Find the least-squares linear approximation to $f(x) = e^x$ over the interval $[-1,1]$.	5	5