STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

B.Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH-I MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCE AND SERIES
SUBJECT CODE : 23MT/MC/SS44
TIME : 3 HOURS **MAX.MARKS: 100**

Q. No.	SECTION A $(5 \times 2 = 10)$ Answer ANY FIVE questions	CO	KL
1.	If $f(x) = 1 + \sin x$ $(-\infty < x < \infty)$ and $g(x) = x^2$ $(0 \le x < \infty)$ then find $g \circ f(x)$.	1	1
2.	Prove that $\lim_{n \to \infty} \frac{n^2}{(n-7)^2 - 6} = 1$.	1	1
3.	Define limit superior with an example.	1	1
4.	What is a Cantor set?	1	1
5.	Define Fourier Series.		
6.	Define absolute convergence and conditional convergence.	1	1
Q. No.	**SECTION B $(10 \times 1 = 10)$** **Answer ALL questions**	**CO**	**KL**
7.	A function $f: A \to B$ is one–one if ____________. a) Every element of B has a preimage b) Distinct elements of A have distinct images c) Every element of A has two images d) $A = B$	2	2
8.	The least upper bound of the sequence $a_n = 1 - 1/n$ is___. a) 0 b) ½ c) 1 d) Does not exist	2	2
9.	A set is countable if__________. a) It is finite only b) It can be put in one–one correspondence with $\mathbb{N}$ c) It is uncountable d) It has no limit points	2	2
10.	A monotone increasing and bounded sequence is__________. a) Divergent b) Convergent c) Oscillatory d) Unbounded	2	2

11.	The sequence $a_n = \frac{n}{n+1}$ is __________. a) Increasing and bounded b) Decreasing and unbounded c) Oscillatory d) Divergent	2	2
12.	The root test is mainly used for ___________. a) Alternating series b) Finite sums c) Bounded sequences d) Power series	2	2
13.	The series $\sum 1/n^2$ is ___________. a) Divergent b) Conditionally convergent c) Absolutely convergent d) Oscillatory	2	2
14.	A series with non-negative terms converges if and only if ____. a) Its sequence of partial sums is bounded b) Terms decrease c) Terms go to zero d) It is alternating	2	2
15.	If f(x) is an even function, its Fourier series contains_______. a) Only sine terms b) Only cosine terms c) Both sine and cosine terms d) No constant term	2	2
16.	Half–range sine series is used for_________. a) Even function b) Bounded sequences c) Constant functions d) Odd functions	2	2
Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
17.	Prove that if A is any nonempty subset of R that is bounded below, then A has a greatest lower bound in R.		
18.	Prove that the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is Cauchy if and only if it is convergent.	3	3
19.	If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that (a) $a_1 \geq a_2 \geq \ldots \geq a_n \geq a_{n+1} \geq \ldots$ and (b) $\lim a_n = 0$ then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is convergent.	3	3
20.	State and prove Ratio test.	3	3

Q. No.	**SECTION D** $(2 \times 15 = 30)$ **Answer ANY TWO questions**	CO	KL
21.	If $f: A \to B$ and $X \subset A, Y \subset A$, then prove that $$f(X \cup Y) = f(X) \cup f(Y).$$	4	4
22.	Show that a) If $0 < x < 1$, then $\{x^n\}_{n=1}^{\infty}$ converges to 0. b) If $1 < x < \infty$, then $\{x^n\}_{n=1}^{\infty}$ diverges to infinity.	4	4
23.	Show that $$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}, (-\pi < x < \pi).$$ Deduce that a) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$ b) $\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$ c)) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$.	4	4
24.	For each $n \epsilon I$, let $I_n = [a_n, b_n]$ be a closed bounded interval of real numbers such that a) $I_1 \supset I_2 \supset \cdots I_n \supset I_{n+1} \supset \cdots$, and b) $\lim_{n\to\infty}(b_n - a_n) = \lim_{n\to\infty}(\text{length of } I_n) = 0$ then prove that $\bigcap_{n=1}^{\infty} I_n$ contains precisely one point.	4	4
Q. No.	**SECTION E** $(2 \times 10 = 20)$ **Answer ANY TWO questions**	**CO**	**KL**
25.	Prove that the countable union of countable sets is countable.	5	5
26.	Express $f(x) = x, (-\pi \leq x \leq \pi)$ as a Fourier series with period 2π.	5	5
27.	Show that a nondecreasing sequence which is bounded above is convergent.	5	5
28.	Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers whose partial sums $s_n = \sum_{k=1}^{n} a_k$ form a bounded sequence, and let $\{b_n\}_{n=1}^{\infty}$ be a nonincreasing sequence of nonnegative numbers which converges to 0, then prove that $\sum_{k=1}^{\infty} a_k b_k$ converges.	5	5