STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2023–24)

B. Sc. DEGREE EXAMINATION, APRIL 2026
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE	**:**	**MAJOR CORE**
PAPER	**:**	**PRINCIPLES OF COMPLEX ANALYSIS**
SUBJECT CODE	**:**	**23MT/MC/CA65**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer any FIVE questions	CO	KL
1.	Define analytic function.	1	1
2.	Define harmonic conjugate of a function.	1	1
3.	Define conformal mapping.	1	1
4.	State Cauchy-Goursat theorem.	1	1
5.	Define zeros of an analytic function.	1	1
6.	State argument principle theorem.	1	1
Q. No.	**SECTION B ($10 \times 1 = 10$)** **Answer ALL questions**	**CO**	**KL**
7.	The real part of the function $w = z^2$ where $z = re^{i\theta}$ is _________________. a) $r^2 \sin 2\theta$ b) $r^2 \sin \theta$ c) $r^2 \cos \theta$ d) $r^2 \cos 2\theta$	2	2
8.	A composition of continuous functions is ______________________. a) differentiable b) analytic c) continuous d) not continuous	2	2
9.	In a two-dimensional fluid flow, the function $\varphi(x, y)$ is called _________________ where $\varphi(x,y) = \int_{(x_0,y_0)}^{(x,y)} p(s,t)\,ds + q(s,t)\,dt.$ a) equipotentials b) stream lines c) velocity potential d) complex potential	2	2
10.	The image of the line segment $y = c, c > 0, -\pi \leq x \leq \pi$ is ____________. a) ellipse b) upper half of ellipse c) circle d) lower half of circle	2	2
11.	The transformation $w = iz - i$ maps the plane $x > 0$ onto the half plane ____. a) $v > 0$ b) $v > -1$ c) $v > 1$ d) $v < 0$	2	2
12.	The minimum value of the function $f(z) = (z+1)^2$ defined on the closed triangular region with vertices at the points $z = 0, z = 2, z = i$ is _________. a) 0 b) $\sqrt{2}$ c) 3 d) 1	2	2

13.	The value of Log (-1) is _________________. a) 0 b) $2\pi i$ c) πi d) $2n\pi$	2	2
14.	The order of the pole at $z = -i$ for the function $f(z) = \frac{z+4}{(z+i)^3\ (z-i)^2}$ is ______. a) 2 b) 3 c) 1 d) 4	2	2
15.	The winding number is ________________ if the image Γ of curve C winds around the origin in the counterclockwise direction. a) positive b) negative c) zero d) none of the above	2	2
16.	The value of $\Delta_C \arg f(z)$ for the function $f(z) = \frac{z^3+2}{z}$ with respect to the circle $\lvert z\rvert = 1$ is ____________________. a) 2π b) 4π c) -2π d) 0	2	2
Q. No.	**SECTION C (2×15 = 30)** **Answer any TWO questions**	**CO**	**KL**
17.	State and prove sufficient conditions for differentiability of a complex valued function.	3	3
18.	a) Compute the region onto which the half plane $y > 0$ is mapped by the transformation $w = (1+i)z$. b) Solve $\int_C \frac{1}{z^2+4}\, dz$ where C is the circle $\lvert z - i\rvert = 2$ in the positive sense, by applying Cauchy integral formula. (7 + 8)	3	3
19.	State and prove Taylor's theorem.	3	3
20.	Apply Cauchy residue theorem to compute the value of the integral $\int_C \frac{4z-5}{z(z-1)}\, dz$ where C is the circle $\lvert z\rvert = 2$ described in the counterclockwise direction.	3	3
Q. No.	**SECTION D (2×15 = 30)** **Answer any TWO questions**	**CO**	**KL**
21.	a) Examine whether the function $f(z) = z^2$ is differentiable everywhere. If yes then find the derivative of the function. b) Investigate the transformations of circles and lines by the mapping $w = \frac{1}{z}$. (5 + 10)	4	4
22.	State and prove Cauchy integral formula.	4	4

23.	a) Obtain the Laurent's series expansion for the function $f(z) = \frac{z+1}{z-1}$ in the domain $1 < \lvert z \rvert < \infty$. b) Categorize the three types of isolated singular points in detail. (7 + 8)	4	4
24.	a) Determine the number of roots, counting multiplicities, of the polynomial $z^4 + 3z^3 + 6 = 0$ inside the circle $\lvert z \rvert = 2$. b) Discuss the mapping of a vertical line segments by the mapping $w = \sin z$. (7 + 8)	4	4
Q. No.	**SECTION E (2×10 = 20)** **Answer any TWO questions**	**CO**	**KL**
25.	If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain *D* then prove that its complement functions *u* and *v* are harmonic in *D*.	5	5
26.	Find the linear fractional transformation that maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto the points $w_1 = 1, w_2 = i, w_3 = -1$.	5	5
27.	State and prove Liouville's theorem.	5	5
28.	Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}, -1 < a < 1$.	5	5

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