STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE : MT/PE/TS34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2008
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : ELECTIVES
PAPER : TENSOR ANALYSIS AND SPECIAL THEORY OF RELATIVITY

TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. a) Determine whether each of the following quantities is a tensor. If so, state whether it is contravariant or covariant and give its rank.
   (i) dx^k (ii) $\frac{\partial \phi}{\partial x^k}\left(x^1, x^2, \ldots x^N\right)$.
   b) Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one.

2. Define metric tensor and conjugate metric tensor. Prove that the angles between the coordinate curves in a three dimensional coordinate system are given by $\cos\theta_{ij} = \frac{g_{ij}}{\sqrt{g_{ii}g_{jj}}}$. Hence show that for an orthogonal coordinate system $g_{ii} = \frac{1}{g^{ii}}$.

3. A quantity $A(p,q,r)$ is such that in the coordinate system x^i, $A(p,q,r)B_r^{qs} = C_p^s$ where B_r^{qs} is an arbitrary tensor and C_p^s is a tensor. Prove that $A(p,q,r)$ is a tensor.

4. Prove that $div\ A^p = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^k}\left(\sqrt{g}A^k\right)$. Hence express $div\ A^p$ in terms of its physical components for spherical polar coordinates.

5. Describe the experiment of Michelson and Morley to determine the motion of the earth with respect to the privileged frame of reference. What was the out come of the experiment?

6. Obtain the effects of Lorentz equations on length and time measurements in different frames of reference.

7. Derive the Euler- Lagrange equation in relativistic analytical mechanics.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) Define Christoffel's symbols of the first and second kind. Obtain their transformation laws and show that they are not tensors.
   b) Evaluate Christoffel's symbols of the first and second kind for spaces where $g_{pq} = 0$ if $p \neq q$.

9. a) Prove the following
   (i) $\dfrac{\partial g_{ij}}{\partial x^k} = [ik, j] + [jk, i]$
   (ii) $\dfrac{\partial g^{ij}}{\partial x^k} = -g^{il}\begin{Bmatrix} j \\ k \quad l \end{Bmatrix} - g^{jl}\begin{Bmatrix} i \\ k \quad l \end{Bmatrix}$
   (iii) $\begin{Bmatrix} i \\ i \quad j \end{Bmatrix} = \dfrac{\partial}{\partial x^j}\left(\log \sqrt{g}\right)$
   b) Prove that the covariant derivation of g_{ij}, g^{ij} and δ^i_j are zero.

10. Define Galilean transformations and show that the fundamental laws of classical mechanics are covariant with respect to these transformations.

11. a) Derive Lorentz transformation equations.
    b) Obtain the relativistic law of addition of velocities.

12. Define relativistic mass of a particle and obtain the formula for it. Hence define relativistic momentum and relativistic kinetic energy.

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