STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09)

SUBJECT CODE: MT/PE/OP14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2008
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : ELECTIVES
PAPER : OPTIMIZATION TECHNIQUES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. Two products X and Y are produced on the three machines M_1, M_2 and M_3. The processing time per unit, availability of machine-hours and the profit per unit are shown below.

Machine	Hours Required		Availability hours
	X	Y	
M_1	2	3	150
M_2	3	2	150
M_3	1	1	100

Profit per units Rs. 4 Rs. 3

Formulate the mathematical model and solve it by simplex method for maximizing the profit.

2. Using big-M method, solve the following LPP.

Minimize: $Z = 2x_1 + x_2$

Subject to: $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

3. Obtain the North-West corner rule and Vogel's approximation solutions to the following transportation problem.

		DESTINATION					
ORIGIN		D_1	D_2	D_3	D_4	D_5	Availability
	O_1	3	5	8	9	11	20
	O_2	5	4	10	7	10	40
	O_3	2	5	8	7	5	30
	Requirement	10	15	25	30	40	

4. An organization producing four different products A,B,C and D has four operators P,Q,R and S, where each operator works effectively for 7 hours a day. The time required for each operator for producing each of the product are given below. The profit on each of the product are given below. The profit on each of the product is also given. Only one operator can be assigned to each of the machine.

	Product			
Operator	A	B	C	D
P	6	10	14	12
Q	7	5	3	4
R	6	7	10	10
S	20	10	15	15
Profit. Rs/Unit	3	2	4	1

Find the assignment of operators to machines which will maximize the profit.

5. Solve the following integer programming problem by Gomory method.

Maximize : $Z = x_1 + x_2$

Subject to : $2x_1 + 5x_2 \le 16$

$6x_1 + 5x_2 \le 30$

$x_1, x_2 \ge 0$ and integer

6. The activity and duration of a project are given below. Draw the network diagram and prepare the table of earliest start time, earliest finish time, lastest start time and latest finish time and also total float.

Activity	Duration
$1 \rightarrow 2$	4
$1 \rightarrow 3$	9
$2 \rightarrow 6$	3
$3 \rightarrow 4$	8
$3 \rightarrow 5$	7
$4 \rightarrow 6$	2
$5 \rightarrow 6$	5

7. A petrol bunk has one cashier who takes on an average 4 minutes to service a customer. Customers come to cashier at random but on an average of 10 people per hour. The management is interested in the following:

(a) What is the average length of the waiting line to be expected under the existing condition?

(b) What proportion of the cashier is idle?

(c) What is the average waiting time of a customer?

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. Apply the principle of duality to solve the following LPP.

Maximize : $Z = 3x_1 + 2x_2$

Subject to : $x_1 + x_2 \le 7$

$x_1 + x_2 \ge 1$

$x_1 + 2x_2 \le 10$

$x_2 \le 3$

$x_1, x_2 \ge 0$

9. Solve the following transportation problem.

Profit (Rs./unit)					
Destination					
Origin	D_1	D_2	D_3	D_4	Supply
O_1	40	25	22	33	100
O_2	44	35	30	30	30
O_3	38	38	28	30	70
Demand	40	20	60	30	

10. Solve the following traveling salesman problem so as to minimize the cost per cycle.

Cost:

		To				
From		A	B	C	D	E
	A	-	3	6	2	3
	B	3	-	5	2	3
	C	6	5	-	6	4
	D	2	2	6	-	6
	E	3	3	4	6	-

11. A local authority intends to carryout a capital project using direct labour. The following table shows the normal time, shortest time and cost of reduction per day. The cot of reduction remains the same per day irrespective of the number of days involved.

Activity	Normal Time (Days)	Shortest Time (Days)	Cost of reduction Per day (Rs.)
1 – 2	6	4	80
1 – 3	8	4	90
1 – 4	5	3	90
2 – 4	3	3	-
2 – 5	5	3	40
3 – 6	12	8	200
4 – 6	8	5	50
5 – 6	6	6	-

The cost of completing the eight activities in normal time is Rs.5800 excluding site overhead. The over headcost of general site activities is Rs.160 per day.

(a) Find the normal duration of the project and its cost and critical path.

(b) Calculate the lowest cost and the associated time.

(c) Calculate the shortest time and the associated cost.

12. a) Describe (M/M/1): (∞/FCFS_) model. With the usual rotation obtain the probability distribution of queue length.

b) Customers arrive in a single-window service station with the mean arrival rate of 10 customers per hour. The service time per customer in exponential distribution with mean of 5 minutes. The counter can accommodate only 3 customers and others have to wait outside.

(i) What is the probability that an arriving customer can directly be served?

(ii) What is the probability that an arriving customer will have to wait outside?

(iii) How long the arriving customer expected to wait before starting service?

(iv) How many customer space should be provided in front of the window so that all arriving customers can wait in front of the window at least 20% of the time?