**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2004 – 05 & thereafter)**

**SUBJECT CODE : MT/PC/TO34**

**M. Sc. DEGREE EXAMINATION, NOVEMBER 2008**
**BRANCH I - MATHEMATICS**
**THIRD SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : TOPOLOGY**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A** **( 5 X 8 = 40 )**

**ANSWER ANY FIVE QUESTIONS**

1. If $E$ is a complete metric space and if $E$ is the union of a sequence of its subsets, show that the closure of at least one set in the sequence must have non empty interior.
2. Let $A$ be an arbitrary non empty subset of a topological space $X$. Show that $x \in \overline{A}$ if and only if every neighbourhood of $x$ intersects $A$.
3. State and prove Heine Borel theorem.
4. Show that a metric space is sequentially compact if and only if it has Bolzone-Weirstrass property.
5. Show that every compact subset of a Hausdorff space is closed.
6. Show that a subspace of the real line R is connected if and only if it is an interval.
7. Show that the product of any non empty class of connected spaces is connected.

**SECTION – B** **( 3 X 20 = 60)**

**ANSWER ANY THREE QUESTIONS**

8. a) Let $X$ and $Y$ be metric spaces and $f$ a mapping of $X$ in $Y$. Show that $f$ is continuous if and only if $x_n \to x \Rightarrow f(x_n) \to f(x)$.
   b) Let $f : X \to Y$ be a mapping of one topological space into another. Show that the following conditions are equivalent.
   (i) $f$ is continuous
   (ii) $f^{-1}(E)$ is closed in $X$ for every closed set $E$ in $Y$.
   (iii) $\bar{f}(\overline{A}) \subset \overline{f(A)}$ for $A \subset X$. (8+12)

9. a) Show that a topological space is compact if every sob basic open cover has a finite subcover.
   b) State and prove Tychonoff's theorem. (12+8)

10. a) State and prove Lebesque covering lemma
    b) Show that every continuous function defined on a compact space is uniformly continuous. (10+10)

11. a) Show that compact Hausdurff space is normal.
    b) State and prove Uryshon Imbedding theorem. (6+14)

12. a) Let $X$ be a compact Hausdorff space. Show that $X$ is totally disconnected if and only if it has an open base whose sets are also closed.
    b) Illustrate by examples that connected spaces need not be locally connected and locally connected spaces need not be connected.