STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2004 – 05 & thereafter)

SUBJECT CODE: MT/PC/CA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2008
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : COMPLEX ANALYSIS – PART II
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 8 = 40)**

ANSWER ANY FIVE QUESTIONS

1. State and prove the mean value property of harmonic functions.
2. Prove that for $\sigma = \text{Re } s > 1$, $\frac{1}{\varsigma(s)} = \prod_{n=1}^{\infty}\left(1 - P_n^{-s}\right)$.
3. Prove that a family $\mathcal{F}$ is totally bounded if and only if to every compact set $E \subset \Omega$ and every $\varepsilon < 0$ it is possible to find $f_1, f_2, ..., f_n \in \mathcal{F}$ such that every $f \in \mathcal{F}$ atisfies $d(f, f_j) < \varepsilon$ on E for some f_j.
4. State and prove Schwarz-Christoffel formula.
5. Prove that a continuous function $u(z)$ which satisfies the mean value property is necessarily harmonic.
6. Prove that any two bases of the same module are connected by a unimodular transformation.
7. Prove that a nonconstant elliptic function has equally many poles as it has zeros.

SECTION – B **(3 X 20 = 60)**

ANSWER ANY THREE QUESTIONS

8. a) State and prove Poisson's integral formula.

   b) Prove the functional equation $\varsigma(s) = 2^s \pi^{s-1} \sin\frac{\pi s}{2}\Gamma(1-s)\varsigma(1-s)$
9. State and prove Arzala's theorem.
10. State and prove Riemann mapping theorem.

11. (a) Prove that $\rho'(z)^2 = 4\rho(z)^3 - g_2\rho(z) - g_3$

(b) Prove that the zeros $a_1,...,a_n$ and the poles $b_1,...,b_n$ of an elliptic function satisfy $a_1 + ... + a_n \equiv b_1 + ... + b_n (\bmod M)$.

12. a) Prove that $\begin{vmatrix} \rho(z) & \rho'(z) & 1 \\ \rho(u) & \rho'(u) & 1 \\ \rho(u+z) & -\rho'(u+z) & 1 \end{vmatrix} = 0$

b) Prove that the function $P_U(z) = \frac{1}{2\pi}\int_0^{2\pi} \operatorname{Re}\frac{e^{i\theta}+z}{e^{i\theta}-z}U(\theta)d\theta$ is harmonic for $|z|<1$, and prove that $\lim\limits_{x \to e^{i\theta_0}} P_U(z) = U(\theta_0)$ provided that U is continuous at θ_0.

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