STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 600 086. (For candidates admitted during the academic year 2023 – 2024 & thereafter)

M.Sc. DEGREE EXAMINATION NOVEMBER 2024 PHYSICS FIRST SEMESTER

| COURSE | : | MAJOR CORE |
|--------------|---|--------------------------|
| PAPER | : | MATHEMATICAL PHYSICS – I |
| SUBJECT CODE | : | 23PH/PC/MP14 |
| TIME | : | 3 HOURS |

MAX. MARKS: 100

| Q. No. | SECTION A | CO | KL |
|----------------|--|------------|----|
| | Answer ALL the questions:(10x 3=30 marks) | | |
| 1 | Enumerate the properties of the operators Δ and E. | CO1 | K1 |
| 2 | Give the values of derivatives of an analytic function of complex variable $z = x + iy$. | CO1 | K1 |
| 3 | Introduce the contravariant and covariant tensors of second rank. | CO1 | K1 |
| 4 | Define isomorphism of vector space. | CO2 | K2 |
| 5 | Show that the process of raising and lowering of indices are reciprocal to each other. | CO2 | K2 |
| 6 | Give the three other forms of Gamma functions. | CO2 | K2 |
| 7 | Define Integrals of trigonometric functions of $\cos\theta$ and $\sin\theta$ and explain the integration around the unit circle. | CO2 | K2 |
| 8 | Give the relation between ket and bra vectors and inner products. | CO3 | K3 |
| 9 | Check whether log z is an analytic function of complex variable $z = x + iy$. | CO3 | K3 |
| 10 | Applying the Legendre polynomial $P_n(x)$, show that $P_{2m+1}(0) = 0$. | CO3 | K3 |
| Q. No. | SECTION B (30 marks) | CO | KL |
| X •1+0• | PART A | | |
| | Answer any TWO questions: $(2x 5 = 10 \text{ marks})$ | | |
| 11 | Transform $ds^2 = dx^2 + dy^2 + dz^2$ in spherical coordinates. | CO3 | K3 |
| 12 | S.T if $f(z) = u + iv$ is an analytic function and $\mathbf{F} = u\mathbf{i} + u\mathbf{j}$ is a vector, then div $\mathbf{F} = 0$ and curl $\mathbf{F} = 0$ are equivalent to Cauchy – Reimann equations. | CO3 | K3 |
| 13 | Prove the recurrence relation, $XJ_n'(x) = xJ_n(x) - X J_{n+1}(X)$ | CO3 | К3 |
| | PART - BAnswer any FOUR questions: $(4x \ 5 = 20 \ marks)$ | | |
| 14 | What is the method of Iteration? Explain the condition for convergence of Iterations. | CO4 | K4 |
| 15 | Use Rodrigue's formula, to find first four Legendre polynomials. | CO4 | K4 |
| 16 | Expand $f(z) = sinz$ into a Taylor series about $z = \pi/4$ | CO4 | K4 |
| 17 | Explain the outer product and inner product of tensors. | CO4 | K4 |
| 18 | Apply vector methods, to derive an equation of heat flow in solids. | CO4 | K4 |

| Q. No. | SECTION C | CO | KL | | |
|--------|--|-----|-----|--|--|
| | Answer the following: (2 x20=40 marks) | | | | |
| 19. | A) Derive the Newton – Gregory formula for forward and | CO5 | K5 | | |
| | backward interpolation. | | | | |
| | B) Explain the Gram Schmidt orthogonalization process and | | | | |
| | use it to construct an orthonormal set of vectors from the set | | | | |
| | $X_1 = (1,2,1) X_2 = (2,1,4) \& X_3 = (4,5,6)$ | | | | |
| | (OR) | | | | |
| | C) Establish the orthogonal properties of Legendre's | CO5 | K5 | | |
| | polynomials. | | | | |
| | D) State and prove Cauchy Integral formula and thereby dz | CO5 | K6 | | |
| | solve the integral $\oint \frac{dz}{(Z^2+Z)}$, where C is a circle defined by | | | | |
| | Z = R > 1 | | | | |
| 20. | $ Z = R > 1 $ A) Show that $A \begin{pmatrix} -xy & -y^2 \\ x^2 & xy \end{pmatrix}$ is a tensor whereas | CO5 | K5 | | |
| | A) Show that $\begin{vmatrix} A \\ x^2 \end{vmatrix}$ is a tensor whereas | | | | |
| | , | | | | |
| | | | | | |
| | $\left[-xy - y^2\right]$ | | | | |
| | $B\begin{pmatrix} -xy & -y^2 \\ x^2 & -xy \end{pmatrix}$ is not a tensor. | | | | |
| | B) Find the numerical solution of $dy/dx = x+y$ from $x = 0$ to | CO5 | K6 | | |
| | 0.2 by Euler's method and modified Euler's method with the | 005 | IX0 | | |
| | initial conditions $x_0 = 0$, $y_0 = 1$. | | | | |
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| | (OR) C) Prove that $\Gamma(2m) = 2^{2m-1}(\pi)^{-1/2} \Gamma m \Gamma(m+1/2)$. Hence | CO5 | K5 | | |
| | show that $(m+1/2)! = \pi^{1/2} (2m+1)!!/2^{m+1}$ | | _ | | |
| | where $(2m+1)!! = 1.3.5(2m-1)(2m+1)$ | | | | |
| | D) Applying tensors deduce Lagrange's equations of motion | CO5 | K6 | | |
| | and there by extend it to a conservative force system. | 202 | | | |
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