STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR ELECTIVE
PAPER : FUZZY SET THEORY AND APPLICATIONS
SUBJECT CODE : 23MT/PE/FT15
TIME : 3 HOURS MAX. MARKS: 100

Q. No.	**SECTION A ($5 \times 2 = 10$)** **Answer ALL questions**	CO	KL
1.	Show that $S(A,B) = \frac{\lvert A \cap B \rvert}{\lvert A \rvert}$	1	1
2.	Define max-min composition and max-product composition.	1	1
3.	If $c(a) = \frac{1}{2}(1 + \cos \pi a)$ is a fuzzy complement, show that it is not involutive.	1	1
4.	Give examples of triangular fuzzy number and trapezoidal fuzzy number.	1	1
5.	Write any two recent applications of fuzzy system.	1	1

Q. No.	**SECTION B ($10 \times 1 = 10$)** **Answer ALL questions**	CO	KL
6.	What is a fuzzy set? a) A set where elements either belong or do not belong. b) A set with a clear boundary between membership and non-membership. c) A set where elements have degrees of membership. d) A set that contains only numeric elements.	2	2
7.	For a fuzzy set $A(x) = \begin{cases} 1 - e^x, x \geq 0 \\ 0, x < 0 \end{cases}$, the height of A is ____. a) 0 b) 1 c) ∞ d) E	2	2
8.	Scalar cardinality of $A = \frac{0.2}{u} + \frac{0.4}{v} + \frac{0.5}{w} + \frac{0.4}{x} + \frac{1}{y}$ is _____. a) 1 b) 1.5 c) 2.5 d) 0	2	2
9.	What is the extension principle in fuzzy set theory? a) A method for calculating the exact membership of elements b) A technique to extend crisp functions to fuzzy functions c) A principle that defines the boundaries of fuzzy sets d) A rule for combining fuzzy relations into a single crisp value	2	2

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10.	What does the "fuzzy product" operation typically refer to in fuzzy set theory? a) The average of two fuzzy sets b) The multiplication of membership values of two fuzzy sets c) The addition of two fuzzy sets d) The minimum of membership values of two fuzzy sets	2	2
11.	Which operation is typically used to determine the fuzzy intersection of two fuzzy sets A and B? a) Addition b) Maximum c) Minimum d) Division	2	2
12.	In fuzzy set theory, the concept of a "fuzzy interval" refers to_____. a) A range of real numbers without any uncertainty. b) A set of fuzzy numbers defined over a specific interval. c) A precise interval with fuzzy boundaries. d) An interval where all elements have full membership	2	2
13.	Which of the following is an example of a linguistic variable? a) Cold, Warm, Hot b) Age, Years, Time c) Temperature, Celsius, Degree d) Time, Seconds, Duration	2	2
14.	When using fuzzy numbers in real-life applications, they are often used to: a) Handle uncertain or imprecise data. b) Replace all traditional mathematics. c) Limit the use of numbers in decision-making. d) Provide exact measurements.	2	2
15.	Which of the following best describes a fuzzy genetic algorithm? a) An algorithm with no randomness. b) An algorithm that uses fuzzy logic in selection processes c) An algorithm that only uses crisp values d) An algorithm based solely on genetic principles	2	2

Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
16.	Using the definitions of $\alpha-$cuts and strong $\alpha-$cuts, show that the following properties hold for $A, B \in \mathcal{F}(X)$ and $\forall \alpha \in [0,1]$ (i) $A \subseteq B$ iff ${}^{\alpha}(A) \subseteq {}^{\alpha}B$ and $A \subseteq B$ iff ${}^{\alpha+}(A) \subseteq {}^{\alpha+}B$ (ii) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ (iii) ${}^{\alpha}(\overline{A}) = {}^{(1-\alpha)+}(\overline{A})$ **(8+4+3)**	3	3

17.	Show that the extension principle is strong cut-worthy but not cut-worthy.	3	3
18.	a) Calculate the standard arithmetic operations $A - B$ and $A * B$ on real numbers for the given by employing the appropriate principle. $A(x) = \begin{cases} 0 & \text{for } x \le -1,\ x > 3 \\ \frac{x+1}{2} & \text{for } -1 \le x \le 1 \\ \frac{3-x}{2} & \text{for } 1 < x \le 3 \end{cases}$ $B(x) = \begin{cases} 0 & \text{for } x \le 1,\ x > 5 \\ \frac{x-1}{2} & \text{for } 1 \le x \le 3 \\ \frac{5-x}{2} & \text{for } 3 < x \le 5 \end{cases}$ b) How do linguistic variables in fuzzy set theory facilitate the representation of uncertainty and enhance decision-making processes across various applications? **(8+7)**	3	3
19.	Discuss the applications of fuzzy system with genetic algorithm	3	3

Q. No.	**SECTION D ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
20.	a) Derive a necessary and sufficient condition for a fuzzy set to be convex. b) Discuss any two types of fuzzy sets. **(7+8)**	4	4
21.	a) State and prove first decomposition theorem b) Show that the fuzzy set $A = \frac{0.2}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5}$ can be represented by its α −cuts. **(8+7)**	4	4
22.	Show that (i) $\lim_{w\to\infty} min\left[1, (a^w + b^w)^{1/w}\right] = max(a, b)$ (ii) $\lim_{w\to\infty} i_w = \lim_{w\to\infty}\left(1 - min\left[1, ((1-a)^w + (1-b)^w)^{1/w}\right]\right)$ $= \min(a, b)$ **(9+6)**	4	4
23.	Prove that (i) MIN [MIN (A, B), C] = MIN [A, MIN (B, C)] (ii) MIN [A, MAX (A, B)] = A where $\mathrm{MIN}(A,B)(z) = \sup_{z=\min(x,y)} \min[A(x), B(y)]$ $\mathrm{MAX}(A,B)(z) = \sup_{z=\max(x,y)} \min[A(x), B(y)]$, A and B are fuzzy numbers **(8+7)**	4	4

Q. No.	**SECTION E ($2 \times 10 = 20$)** **Answer ANY TWO questions**	CO	KL
24.	Let $A(x) = \begin{cases} 1, & x < 20 \\ \frac{35-x}{15}, & 20 < x < 35 \\ 0, & x \geq 35 \end{cases}$ and $B(x) = \begin{cases} 0, & x \leq 20, x \geq 60 \\ \frac{x-20}{15}, & 20 < x < 35 \\ \frac{60-x}{15}, & 45 < x < 60 \\ 1, & 35 \leq x \leq 45 \end{cases}$ Represent the fuzzy sets $A, B, A \cup B, A \cap B, \overline{A}$, diagrammatically and obtain the membership function for $A \cup B, A \cap B, \overline{A}$.	5	5
25.	Solve the fuzzy relation equation $\mathbf{p} \circ \begin{bmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} = [0.6 \quad 0.6 \quad 0.5]$	5	5
26.	For all $a, b \in [0,1]$, Prove that $u(a, b) \leq u_{max}(a, b)$.	5	5
27.	In what ways does the integration of fuzzy systems into neural networks improve their performance in handling uncertain data, and how can this synergy be assessed in practical applications?	5	5