STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**MAJOR CORE**	
PAPER	**:**	**REAL ANALYSIS**	
SUBJECT CODE	**:**	**23MT/PC/RA14**	
TIME	**:**	**3 HOURS**	**MAX. MARKS: 100**

Q. No.	**SECTION A ($5 \times 2 = 10$)** **Answer ALL questions**	**CO**	**KL**
1.	Define adherent point of a subset of R^n and give an example.	1	1
2.	Give an example of a function which is of bounded variation.	1	1
3.	What is meant by matrix of a linear function?	1	1
4.	Find an example to show the existence of Riemann-Stieltjes integral can be altered by changing the value of f at a single point.	1	1
5.	Define Jacobian determinant and obtain the Jacobian determinant of a complex-valued function.	1	1

Q. No.	**SECTION B ($10 \times 1 = 10$)** **Answer ALL questions**	**CO**	**KL**
6.	The function f is said to be differentiable at c if there exists a linear function $T_c: R^n \to R^m$ such that ________ (a) $f(c+v) = f(c) - T_c(v) + \|v\|E_c(v)$ (b) $f(c+v) = f(c) + T_c(v) + \|v\|E_c(v)$ (c) $f(c+v) = f(c) + T_c(v) - \|v\|E_c(v)$	2	2
7.	If x is an accumulation point of S in $\mathbb{R}^n$, then every $n-$ ball $B(x)$ contains ___________ points of S. (a) finitely many (b) infinitely many (c) no	2	2
8.	The set of rational numbers has every ____________ as an accumulation point. (a) irrational number (b) real number (c) rational number	2	2
9.	The function f satisfies _____________ condition if $\|f(y) - f(x)\| \leq A\|y - x\|$. (a) Lipschitz (b) Riemann (c) Liouville's	2	2
10.	If f is differentiable at a and if $\nabla f(a) = 0$, the point a is called a _________ point of f. (a) Saddle (b) stationary (c) unique	2	2

11.	The real line R^1 is covered by the collection of all __________. (a) open intervals (a, b) (b) closed intervals [a, b] (c) half open intervals.	2	2
12.	Boundedness of f' is not necessary for *f* to be of ___________. (a) bounded variation (b) total variation (c) Riemann integrable	2	2
13.	If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on [a,b] then ___________ on [a,b]. $(a)\ c_1 f + c_2 g \not\subset R(\alpha)$ (b) $c_1 f + c_2 g \notin R(\alpha)$ (c) $c_1 f + c_2 g \in R(\alpha)$	2	2
14.	___________ provide connecting link between Riemann-Stieltjes integrals and finite sums. (a) Step functions (b) Euler's summation formula (c) none of these.	2	2
15.	If *f* is differentiable at *c* then *f* is _____________at *c*. (a) Not continuous (b) continuous (c) not zero	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
16.	a) Let *A* be a subset of R^n and let *F* be an open covering of *A*. Prove that there is a countable subcollection of *F* which also covers *A*. b) Let *f* be defined on [*a*, *b*]. Then prove that *f* is of bounded variation on [*a*, *b*] if, and only if, *f* can be expressed as the difference of two increasing functions. (8+7)	3	3
17.	State and prove Bolzano-Weierstrass theorem.	3	3
18.	a) State and prove chain rule in terms of total derivatives. b) Relating the total derivative to directional derivative prove that the total derivative is unique if it exists. (10+5)	3	3
19.	a) State and prove first mean-value theorem for Riemann-Stieltjes integrals. b) Show that continuous partial derivatives is locally one- to-one near a point where the Jacobian determinant does not vanish. (7+8)	3	3

Q. No.	**SECTION D** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	CO	KL
20.	Analyze the equivalence conditions of the following statements for a subset S of R^n: (i) S is compact. (ii) S is closed and bounded. (iii) Every infinite subset of S has an accumulation point in S.	4	4
21.	a) Prove that if f and g are each of bounded variation on [a, b], then their sum and product are also of bounded variation. b) State and prove the second-derivative test for extrema. (5+10)	4	4
22.	a) Assume that $\alpha \nearrow$ on $[a, b]$, if P' is finer than P, then prove that $U(P', f, \alpha) \leq U(P, f, \alpha)$. b) State and prove the formula for integration by parts in Riemann- Stieltjes Integral. (5+10)	4	4
23.	a) Obtain Taylor's Formula. b) Analyze the consequences of the additive property on the study of the total variation as a function of the right end point. (8+7)	4	4

Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	CO	KL
24.	Evaluate whether the function $f(x) = x \cos\left(\frac{\pi}{2x}\right), x \neq 0$ and $f(x) = 0,\ x = 0$ over the interval [0, 1] is of bounded variation on [0, 1].	5	5
25.	Illustrate with an example the matrix form of chain rule.	5	5
26.	State and prove the inverse function theorem	5	5
27.	Assume α is of bounded variation on $[a, b]$. Let $V(x)$ denote the total variation of α on $[a, x]$ if $a < x \leq b$, and let $V(a) = 0$. Let f be defined and bounded on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then prove that $f \in R(V)$ on $[a, b]$.	5	5

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