**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86**
**(For candidates admitted from the academic year 2023 – 2024 and thereafter)**

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2024**
**BRANCH I - MATHEMATICS**
**THIRD SEMESTER**

**COURSE : MAJOR CORE**
**PAPER : PARTIAL DIFFERENTIAL EQUATIONS**
**SUBJECT CODE : 23MT/PC/PD34**
**TIME : 3 HOURS MAX. MARKS: 100**

| Q. No. | **SECTION A** ($\mathbf{5 \times 2 = 10}$) **Answer ALL questions** | CO | KL |
|---|---|---|---|
| 1. | Discuss the classification of the second order PDE. | 1 | 1 |
| 2. | What is a Neumann boundary condition? | 1 | 1 |
| 3. | What is the physical significance of the diffusion equation? | 1 | 1 |
| 4. | What are the initial conditions for solving a vibrating string problem? | 1 | 1 |
| 5. | Give the physical interpretation of Green's function. | 1 | 1 |

| Q. No. | **SECTION B** ($\mathbf{10 \times 1 = 10}$) **Answer ALL questions** | CO | KL |
|---|---|---|---|
| 6. | The equation $x^2(y-1)z_{xx} - x(y^2-1)z_{xy} + y(y^2-1)z_{yy} + z_x = 0$ is hyperbolic in the entire $xy-$ plane except along<br>(a) $x-$axis (b) a line parallel to $y-$axis<br>(c) $y-$axis (d) a line parallel to $x-$axis | 2 | 2 |
| 7. | Which of the following methods is used to find integral surfaces passing through a given curve?<br>(a) Method of Characteristics (b) Charpit's Method<br>(c) Separation of Variables (d) Laplace Transform | 2 | 2 |
| 8. | The complete integral of the PDE $z = px + qy - \sin(pq)$ is _____<br>(a) $z = ax + by + \sin(ab)$ (b) $z = ax + by - \sin(ab)$<br>(c) $z = ax + y + \sin(a)$ (d) $x + by - \sin(a)$ | 2 | 2 |
| 9. | The solution of the initial value problem $u_{tt} = 4u_{xx}, t > 0, -\infty < x < \infty$ satisfying the conditions $u(x,0) = x, u_t(x,0) = 0$ is<br>(a) $x$ (b) $\frac{x^2}{2}$ (c) $2x$ (d) $2t$ | 2 | 2 |
| 10. | The Cauchy problem for first-order PDEs typically involves which of the following conditions?<br>(a) Dirichlet condition<br>(b) Neumann condition<br>(c) Initial condition along a characteristic curve<br>(d) None of the above | 2 | 2 |
| 11. | Which of the following equations is classified as elliptic?<br>(a) Laplace Equation (b) Heat Equation<br>(c) Wave Equation (d) Schrodinger Equation | 2 | 2 |
| 12. | In solving elliptic PDEs using the method of separation of variables, which assumption is commonly made about the solution?<br>(a) It is a product of functions, each depending on only one variable<br>(b) It is constant<br>(c) It satisfies the Neumann boundary condition<br>(d) None of the above | 2 | 2 |

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| | | | |
|---|---|---|---|
| 13. | Which of the following is the delta function used in the solution of diffusion equations?<br>(a) Green's Function (b) Heaviside Delta Function<br>(c) Dirac Delta Function (d) Bessel Function | 2 | 2 |
| 14. | D'Alembert's solution is specifically used for which of the following equations?<br>(a) Laplace Equation (b) Heat Equation<br>(c) Wave Equation (d) Diffusion Equation | 2 | 2 |
| 15. | The method of images is applied to solve PDEs for which type of boundary conditions?<br>(a) Periodic (b) Dirichlet (c) Mixed (d) Robin | 2 | 2 |

| **Q. No.** | **SECTION C ($\mathbf{2 \times 15 = 30}$)**<br>**Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 16. | Find the integral surface of the linear PDE which contains the circle defined by $x^2 + y^2 + z^2 = 4$, $x + y + z = 2$. | 3 | 3 |
| 17. | The ends $A$ and $B$ of a rod, 10cm in length, are kept at temperatures $0°C$ and $100°C$ until the steady state condition prevails. Suddenly the temperature at the end $A$ is increased to $20°C$, and the end $B$ is decreased to $60°C$. Find the temperature distribution in the rod at time $t$. | 3 | 3 |
| 18. | Derive the one-dimensional wave equation and also obtain its periodic solution. | 3 | 3 |
| 19. | Determine the Green's function for the Helmholtz equation for the half-space $z \geq 0$. | 3 | 3 |

| **Q. No.** | **SECTION D ($\mathbf{2 \times 15 = 30}$)**<br>**Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 20. | Show that the PDEs $xp - yq = x$ and $x^2p + q = xz$ are compatible and hence find their solution. | 4 | 4 |
| 21. | Define and solve the interior Dirichlet problem for a circle. | 4 | 4 |
| 22. | Discuss the solution of non-homogeneous equation of forced vibrations of a finite string due to an external driving force. | 4 | 4 |
| 23. | Use Green's function technique to solve the heat conduction equation with no sources present. | 4 | 4 |

| **Q. No.** | **SECTION E ($\mathbf{2 \times 10 = 20}$)**<br>**Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 24. | Use Charpit's method to solve $p^2x + q^2y = z$. | 5 | 5 |
| 25. | Derive the Laplace equation. | 5 | 5 |
| 26. | An infinite one-dimensional solid defined by $-\infty < x < \infty$ is maintained at zero temperature initially. There is a heat source of strength $g_s(t)$ units, situated at $x = \xi$, which releases constant heat continuously for $t > 0$. Find the expression for the temperature distribution in the solid for $t > 0$. | 5 | 5 |
| 27. | Show that the Green's function $G(r, r')$ has the symmetric property. | 5 | 5 |